WELCOME TO Introduction to Finance for MSc Financial Technology and Innovation

Lecture 4

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Quick recap: We did a lot last lecture, most importantly:

- Annuity (PV, FV)
- Perpetuity (PV, FV, constant growth)
- Interest rate (aka cost of lending) vs rate of return (what I expect from an investment)
- Introducing the following terms: Bond and yield,

We finish up the basic mechanics with:

- PV payments due or PV payments starting in one time period
- Rates: Simple and compound interest, APR, EAR

To understand the conceptual underpinning of a financial problem that deals with series of payments, ask yourself the following questions:

- Is the series of payments for a fixed or for an infinite period?
  - Annuities: I understand that is it for fixed period
  - Perpetuities: I understand that is it forever
- Do I want to calculate the value effects of payments for today or for the future?
  - PV of annuities: what is the value of all fixed payments *today*?
  - PV of perpetuities: what is the value of all occurring payments today? I figure, depreciation plays a role
  - FV of annuities: what will be the value of the series of payments, once it is over?
  - FV of perpetuities: Makes no sense to calculate
- Is there a payment that is occurring immediately, or would the first payment / money flow occur at the end of the given time-period?
  - Ordinary series of payments (ordinary payment)
  - Payment due

Let us return to the Present Value of Perpetuity

Example

What is the cost, aka PV of \$1,000 every year, for all eternity, if you estimate the perpetual discount rate to be 10%? The first payment will take place one year from today.

$$\boxed{PV = \frac{1{,}000 \text{ USD}}{0.10} = 10{,}000 \text{ USD}}$$

$$\boxed{PV_0 = \frac{C_1}{r}}$$

The catch (?):

- Inflation rate
- Interest rate
- It is an infinite promise! So, perpetuities are no market instruments, but the concept is important to value equities

Series of payments: Timing of payment matters!

$$PV_0 = \frac{C_1}{r}$$

$$\text{PV of annuity}_t\text{:}\ C \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^t}\right]$$

It is very important to pay attention to when the first payment is being made

What is the **PV** of a payment that takes place **today**?

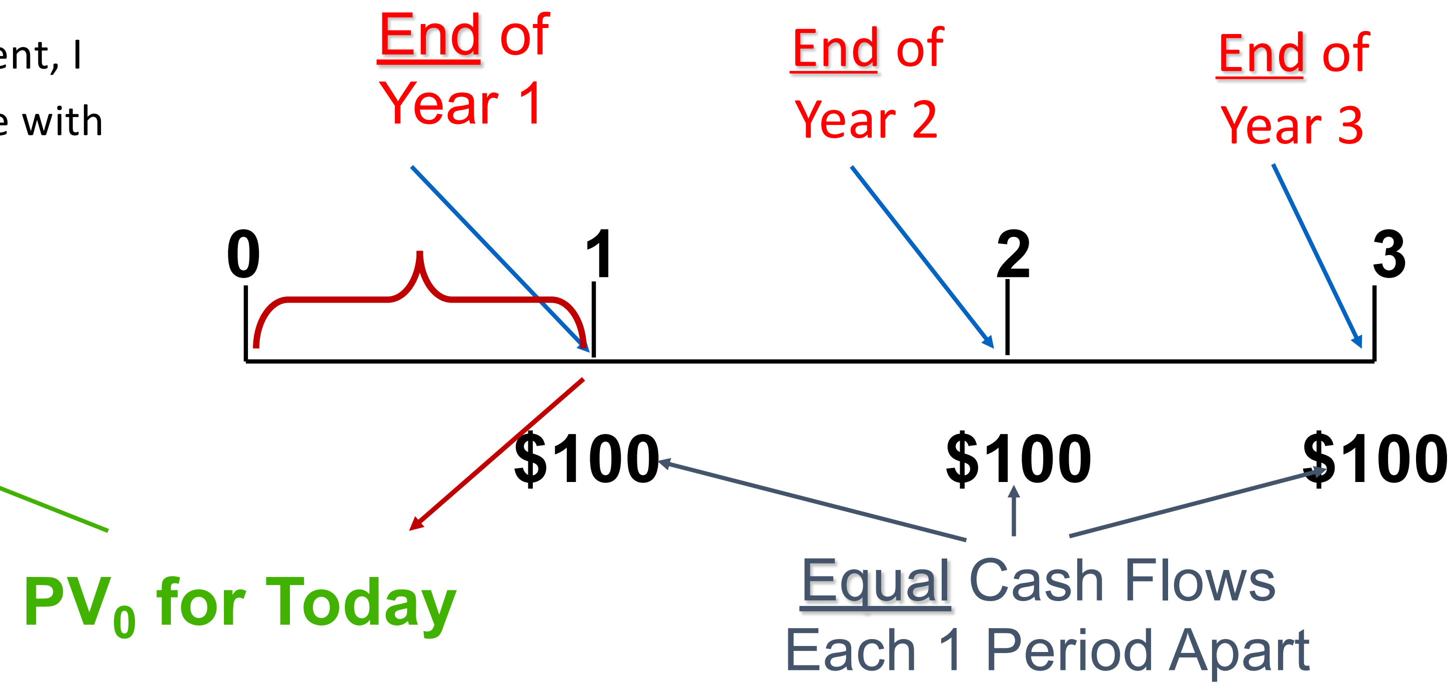
- The **cost** of payment is known, it is happening today, so equals to PV_0

Payments made at the **end** of each period means:

Assume I have three payments of \$100 = C_1, C_2 and $C_{3;}$ and each payment is being made at the end of each year, payment schedule/profile:

After the first payment, I can already calculate with the following costs:

$$\boxed{PV_0 = \frac{C_1}{(1+r)}}$$



Payments made at the **beginning** of each period means:

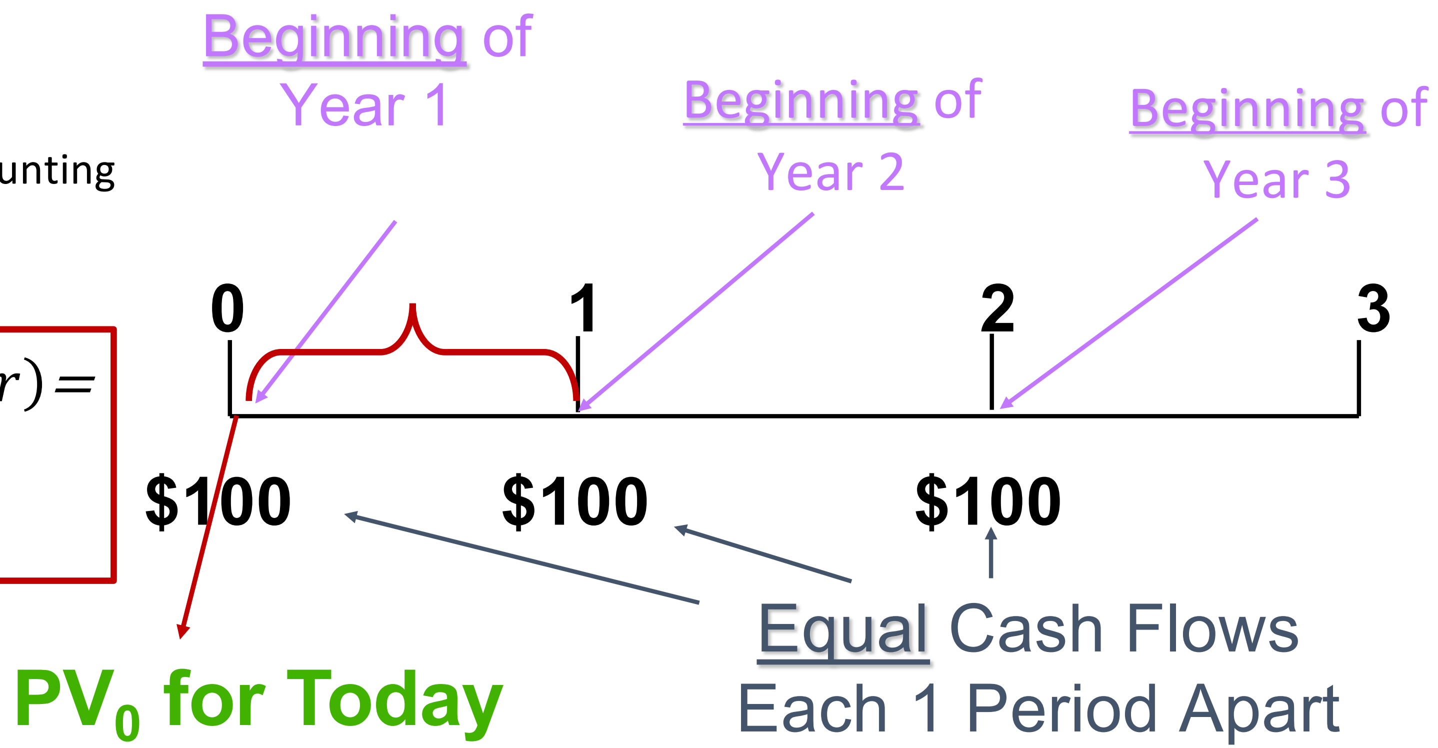
Assume I have three payments of \$100 = C_1, C_2 and C_3; and each payment is being made at the **beginning** of each year, payment profile:

In fact:

I need to adjust for discounting

by one period less

$$\boxed{PV_0 = \frac{C_1}{(1+r)} \times (1+r) = \; = C_1 = 100}$$



Payments made at the **beginning** of each period means:
Adjusting for payments by discounting one period less

$$PV = \frac{C_t}{(1+r)^t} \times (1+r) = \frac{C_t}{(1+r)^{t-1}}$$

PV with payment at the end of the period

Adjustment for discounting by one period less

In sum: Perpetuity with (1) first payment in one payment period of time vs (2) today

$$PV_0 = \frac{C_1}{r}$$

versus

$$PV_0 = \frac{C_1}{r} \times (1+r)$$

PV with first payment in one year/**at the end** of each year

PV with first payment today/ at t=0/ **at the beginning** of each payment period

Adjustment for discounting by one year less

Where

- C_1 is the cash flow to be received each year
- PV_0 is the present value and
- r is the discount rate

Same conceptual design: Annuity with (1) first payment in one payment period of time vs (2) today

$$\text{PV of annuity} = C_1 \times \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

PV with first payment in one year/at the end of each year

$$\underset{\textbf{due}}{\text{PV of annuity}} = C_1 \times \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right] \times (1+r)$$

PV with first payment today, i.e. at t=0, at the beginning of each payment period

Adjustment for discounting by one year less

Also quoted as
Annuity due

Annual Percentage Rate (APR) tells you the interest rate you need to account for (calculate with) per year

However, fact: Interest can be paid **more frequently** than once every year

- For example, we could receive interest semiannually (i.e. once every six months)

We introduce therefore the term **annual interest rate, or annual percentage rate** (APR) to be able to calculate on a yearly basis

Say we have \$100 to invest and we can receive interest **semiannually**.

- The annual interest rate or annual percentage rate (APR) is 10%
- In practice, we are getting 5% = 10% divided by 2; every six months

Interest #1: Simple interest

With simple interest payments

- The first interest payment is made at the end of the first six-month period

At the end of the first payment period, i.e. after 6 months, investment will be worth:

$$FV = \$100 \times (1+0.05) = \$105$$

During the following six months we **ONLY earn interest** on the initial investment of $100

At the end of the second payment period our investment will be worth:

$$FV = \$105 + \$100 \times (0.05) = \mathbf{\$110}$$

Interest #2: Compound interest

Fact: Interest can be paid **more frequently** than once every year

- For example, we could receive interest semiannually (i.e. once every six months)

By compounding we are **earning interest on BOTH**:

- the initial investment
- the interest payments that we receive at the end of each period

Say we have $100 to invest and we can receive interest semiannually.

- The annual interest rate or annual percentage rate (APR) is 10%
- In practice, we are getting 5% = 10% divided by 2; every six months

Interest #2: Compound interest

We start again with a simple interest payment for the first period

- The first interest payment is made at the end of the first six-month period

At the end of the first payment period my investment will be worth:

$$FV = \$100 \times (1+0.05) = \$105$$

During the following six months we will earn interest **on the initial investment of $100 AND the $5 interest that we have received**, i.e. we will earn interest on $105

At the end of the second payment period our investment will be worth:

$$FV = \$105 \times (1+0.05) = \mathbf{\$110.25}$$

Time to talk about **interest rates**.
How is interest paid and quoted?

Annual Percentage Rate (APR)

Interest rate that is annualized
using **simple** interest

Effective Annual Interest Rate (EAR)

Interest rate that is annualized
using **compound** interest

We basically calculate the **percentage change** in our investment over one year

- $100 grew to $110.25 by the end of the year
- Semi-annual payments and compound interest

EAR: **Percentage change between the beginning balance and ending balance**

Looks familiar?
What does it resemble?

$$\boxed{EAR = \frac{\$110.25 - \$100}{\$100} = 0.1025 = 10.25\%}$$

APR (annual % rate) versus EAR (effective annual rate)

APR	Periods per year	Interest per Period (the interest that we receive in practice)	Future Value after one year	EAR
6%	1	6%	$100x(1+0.06)^{1}=106$	$\frac{106-100}{100}=6.000\%$
6%	2	6% / 2 = 3%	$100x(1+0.03)^{2}=106.09$	$\frac{106.09-100}{100}=6.090\%$
6%	4	6% / 4 = 1.5%	$100x(1+0.015)^{4}=106.136$	$\frac{106.136-100}{100}=6.136\%$
6%	12	6% / 12 = 0.5%	$100x(1+0.005)^{12}=106.168$	$\frac{106.168-100}{100}=6.168\%$
6%	52	6% / 52 = 0.1154%	$100x(1+0.001154)^{52}=106.18$	$\frac{106.18-100}{100}=6.180\%$
6%	365	6% / 365 = 0.0164%	$100x(1+0.000164)^{365}=106.184$	$\frac{106.184-100}{100}=6.184\%$

When interest is paid **once a year**, APR and EAR are the **same**

When interest is paid **more frequently**, the EAR is higher than the APR

Your toolset so far:

$$\mathrm{PV} = \underbrace{\frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots + \frac{C_t}{(1+r)^T}}_{\textbf{DCF}}$$

- PV, FV
- Discount factor
- NPV, DCF
- Rate of Return rule
- Interest rates: simple or compounding

$$\mathrm{NPV} = C_0 + \sum_{t-1}^{T} \frac{C_t}{(1+r)^t}$$

$$\mathrm{NPV} = \mathrm{PV} - \text{required investment}$$

$$\text{Return} = \frac{\text{profit}}{\text{investment}}$$

<u>Annual Percentage Rate (APR)</u>

$$\mathrm{APR} = \text{rate per period} \times \text{periods per year}$$

<u>Effective Annual Interest Rate (EAR)</u>

$$\mathrm{EAR} = (1 + \text{rate per period})^{\text{periods per year}} - 1$$

$EAR = [1 + \frac{r}{m}]^m - 1$, where

$\boldsymbol{m}$ = period; $\boldsymbol{r}$ = rate

Your toolset so far:

- Annuity

$$\text{PV of annuity}_t\text{: } C\times\left[\frac{1}{r}-\frac{1}{r\times(1+r)^t}\right]$$

$$\text{FV of annuity} = C\times\left[\frac{(1+r)^t-1}{r}\right]$$

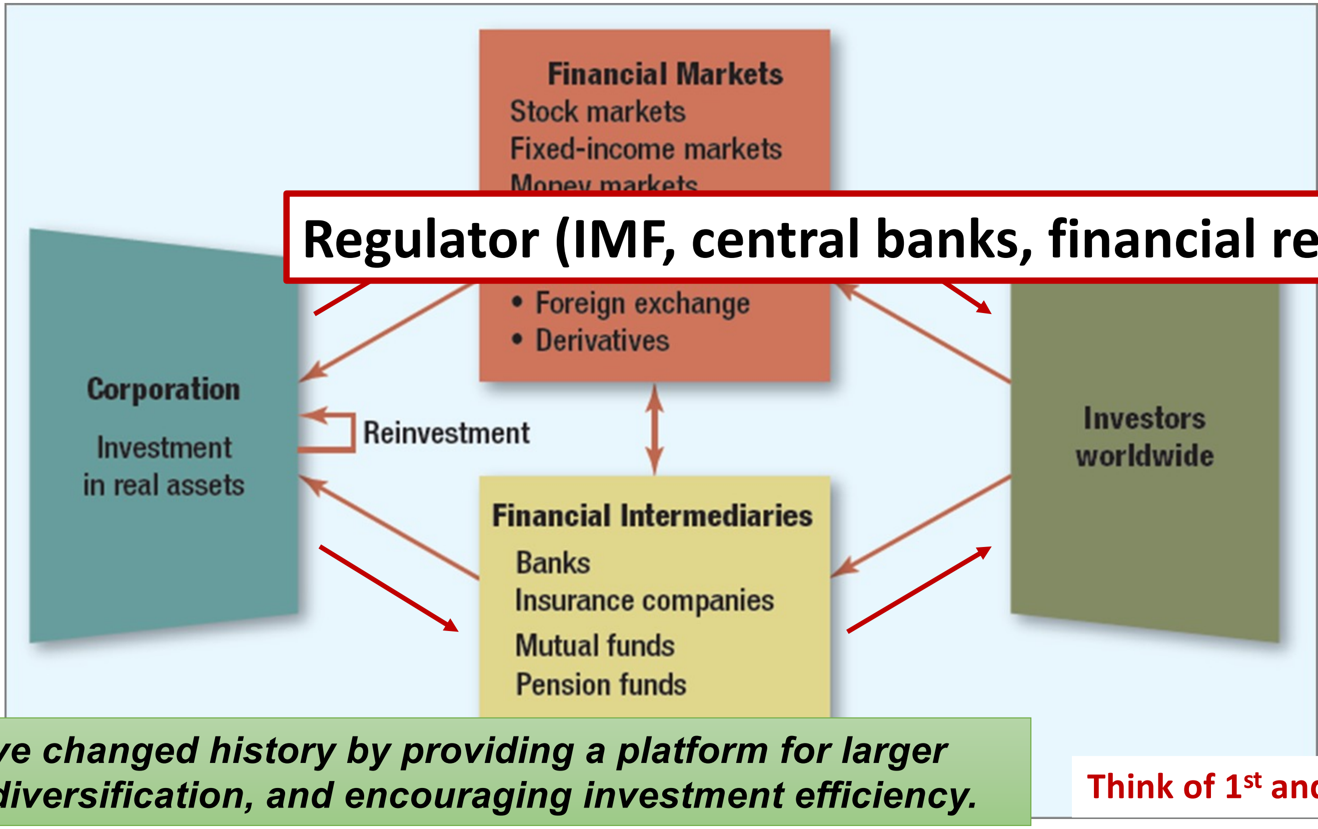
$$\boxed{\underset{due}{\text{PV of annuity}} = C_1\times\left[\frac{1}{r}-\frac{1}{r(1+r)^t}\right]\times(1+r)}$$

- Perpetuity

$$\text{PV of perpetuity} = \frac{C_1}{r}$$

$$\boxed{\text{PV of perpetuity}_{due} = \frac{C_1}{r}\times(1+r)}$$

Capital markets are part of the **Financial System** to channel funds from investors to companies in forms of equity or debt



Regulator (IMF, central banks, financial regulators)

Markets have changed history by providing a platform for larger businesses, diversification, and encouraging investment efficiency.

Think of 1st and 2nd Axiom!

Financial Markets: Any marketplace where trading takes place

Markets have changed history by providing a platform for larger businesses, diversification, and encouraging investment efficiency.

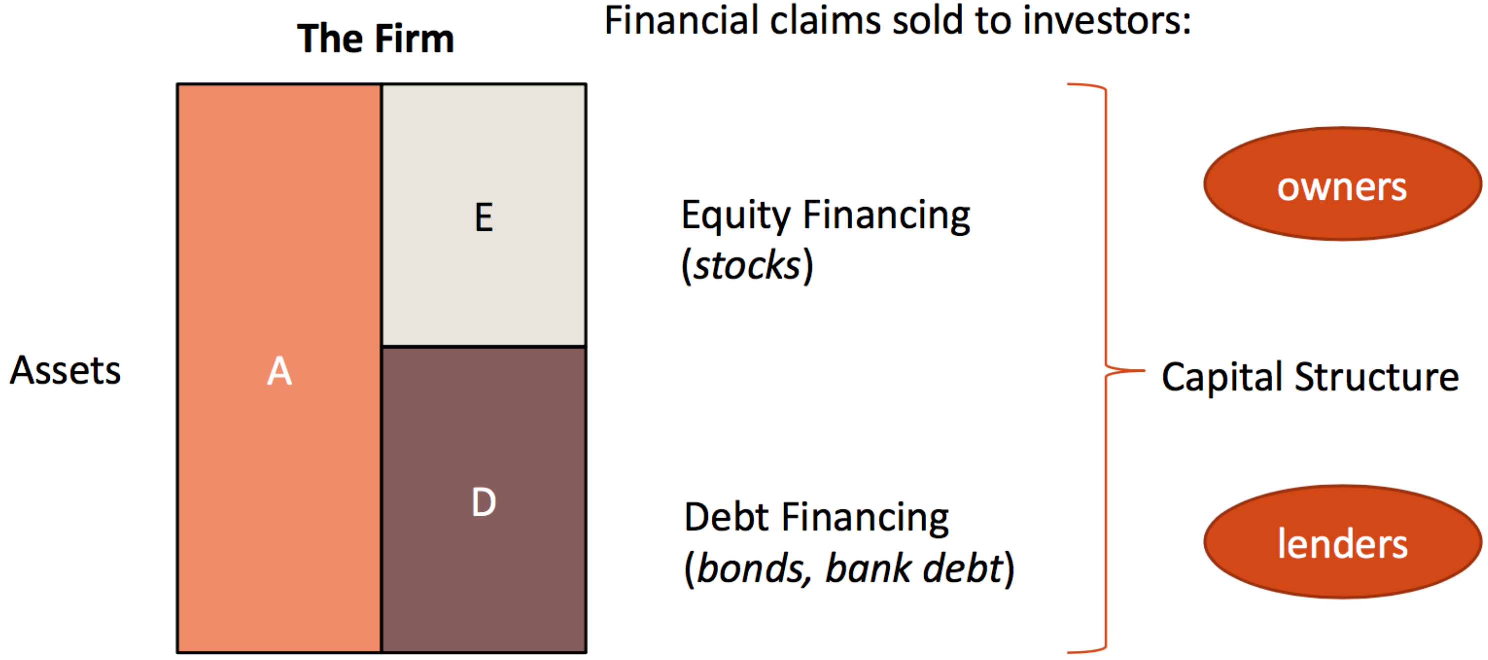
Trading of **financial securities**, i.e. the **financial assets** whose value is derived from a contractual claim

- Remember: financial assets are financial claims on cash flows, a promise to keep!
- We discuss bonds and stocks in this module

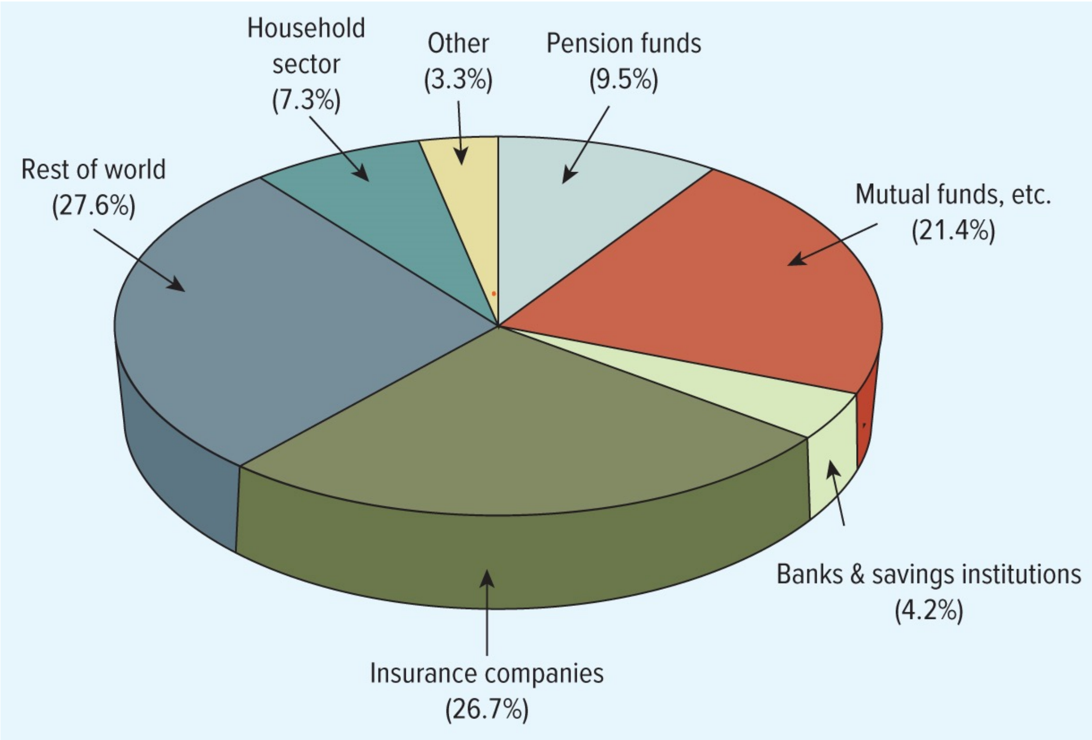
Markets have different functionalities for corporations:

- **Source of funding**
  - Companies can issue stocks and bonds (equity and debt financing)
- **Investor liquidity**
  - Investors can easily buy and sell financial securities
- **Risk management**
  - Investors can include different securities in their portfolio
- **Source of information** (Principle 5!)
  - Company share prices, interest rates, inflation rates

Markets as source of funding
Corporations raise money via financial claims



Financial intermediaries are the biggest **debtholders** in the United States, i.e., ***lenders*** of corporations (December 2020)



Debt is a **prior claim** on cash-flows

- **Bonds** (or notes) are traded on financial markets
- **Loans** are issued by banks directly

As a company, you decide:

1: Borrowing short-term or long-term?

- Bank loan versus bond issuance

2: Fixed or *floating* (variable) interest rate?

- Risk-apetite? Hurdle rate you offer?

Would you lend me money? I need 100 GBP, so I issue you a bond

When the bond matures in 6 months, in April 2025, I guarantee you the 100 GBP.

- Would you buy my bond?
- How much would you pay?

How to make my offer more attractive?

Would you lend me money? I need 1000 GBP, so I issue you a bond, and I pay it back in four years

So, if you want to **buy** my bond, how much would you pay **today**?

- What would it depend on?

We need to address e.g. the opportunity cost of capital. What is my “comparable”?

… let us assume that my comparable rate *r* = 5% (a bit above basic UK interest rate)

PV = ? $= \frac{1000}{(1+0.05)^4} = 819.67$

You would pay me 819.67 GBP? That is not enough. Do you have an advice, how I would get more?

Each year, until my bond ***matures***, I am paying you an interest payment of 3,5%, that is calculated on the capital I am trying to raise.

Let us still assume that my comparable rate is still r = 5%

What would be the money I could get now? How would the equation change, in fact?

$$PV_{old} = \frac{1000}{(1+0.05)^4} = 819.67$$

$$PV_{new} = \frac{35}{(1+0.05)} + \frac{35}{(1+0.05)^2} + \frac{35}{(1+0.05)^3} + \frac{1000+35}{(1+0.05)^4} = 943.67$$

What is a bond?

It is a ***promise***, a financial claim that mirrors the perception over the bond issuer's financial health, economic value.

This is a **contractual obligation**; it represents a claim on a cash flow, and it is worth ***something*** today.

If you hold onto a bond, you are entitled to *receive annual payments*, i.e. a fixed set of cash payoffs based on the ***coupon rate*** that the promise includes, so it is a form of *annuity*.

At maturity, you receive the final interest payment, as well as the ***face value*** of the bond.

Is it a bond?

- I owe you £100; I commit to paying £100 in March 2025
- ***Signed: Zsofia Kraeussl***

This note is an example of the simplest form of a bond, it’s called an IOU

- I commit formally to make a payment
- The note needs to have a date on it

It needs to specify how much I commit to pay in the future, how often, is it only once, will there be any interest payments.

It needs to specify the name of the borrower

Bond becomes a debt for the issuer, thus, it is a liability: The issuer promises something

Investor **A** can buy the note from me and becomes the owner of my debt

- To Investor **A**, my bond is an asset (a financial asset/security)
- Investor A has two options:
    - Option 1: Can **wait** until ***maturity*** and collect the annual interest payments + end payment
    - Option 2: Can **sell** the note to another investor (Investor B) in the bond market and get a payment for the note before the payment date
        - Given Option 2: In a year’s time I will owe the money to Investor B

In sum: Bond is a financial instrument that allows companies to borrow money

The **largest** issuers of bonds are governments.

Digest the terminology: Bonds

- **Bond**
  - Security that obligates the issuer to make specified payments (== interest) to the bondholder.
- **Face value** (par value or principal value or redemption value)
  - Payment at the maturity of the bond.

- **Coupon**
  - The interest payments made to the bondholder.

- **Coupon rate**
  - Annual interest payment, as a percentage of face value.

Time to Maturity: the time it takes for the bondholder to receive the bond’s face value/ length of the contract.

Yield to Maturity (YTM): the rate of return that the investor gets **over the life of the bond**
Remember: RoR is a percentual difference over a timeframe, but time itself is not specificed!

Digest the terminology: Price of a bond today

The price of a bond is the **present value** of all cash flows generated by the bond (that is, coupons and face value) discounted at the required rate of return.

PV (bond) = PV (annuity of coupon payments) + PV (final payment)

$$\mathrm{PV} = \frac{\mathrm{cpn}}{(1+r)^1} + \frac{\mathrm{cpn}}{(1+r)^2} + \ldots + \frac{(\mathrm{cpn} + \mathrm{par})}{(1+r)^t}$$

Note: “cpn” is commonly used as an abbreviation for “coupon”

Again: what is *r* standing for?

For an Investor, who buys the bond: It is the r*equired rate of return*
For an issuer: Interest rate that is paid on a loan to be received (see last lecture’s notes!) == coupon rate!

What is our biggest concern with the rate of return?

It can vary over time!

Investors consider the PV of a bond that is held to its maturity

In October 2022, you purchase 100 euros of bonds in France that pay a **4.25%** coupon every year. If the bond matures in 2026, I *keep my bond until maturity*, so *I know* that the **YTM is 0.15%,** what is the value of the bond?

$$\mathrm{PV} = \frac{\mathrm{cpn}}{(1+r)^1} + \frac{\mathrm{cpn}}{(1+r)^2} + \ldots + \frac{(\mathrm{cpn} + \mathrm{par})}{(1+r)^t}$$

$$\mathrm{PV} = \frac{€4.25}{1.0015} + \frac{4.25}{(1.0015)^2} + \frac{4.25}{(1.0015)^3} + \frac{104.25}{(1.0015)^4} = €116.34$$

What mathematics tells us is the *lower* is the YTM, the *higher* is the price of a bond.

Thus: The *higher* is the YTM, the *lower* is the price of a bond.

What does it mean for Finance? How would you explain this dynamics? Hint: Think of principles 1, 2, and 4 combined

“The *higher* is the rate of return (or YTM over the lifetime of the bond), the *lower* is the price of a bond.”

Market rate can vary, so opportunity costs can. However, coupon rate and principal are not affected: they are given by the financial contract!

In this case: If (e.g. market risk, so) market rate increases, investors would prefer paying less price for a higher risk they take.

On the other hand, if rates decrease, investors not only receive annual coupon payments, but can potentially realize **a capital gain** between the face value and the price.

Why (again)?

Investors are concerned about the yield (return) they get for the risk they take!

Bond yields or returns to investors are always stated as a ***percentage*** (also known as the <u>cost to the issuer</u>) = the YTM

Usually, bonds are **quoted** or offered in ***spreads to an index*** (and **not** on price), *so we say that BMW (bond) is trading at 220* ***over Bunds*** (here, Bunds is the index, the reference)

- *What is an index?: A mathematical measure to track performance of investments*

In other words:

"BMW's bonds have a YTM of 2.20% over the equivalent maturity (time) German Government Bond (Bund)."

We always state spreads in terms of ***BASIS POINTS*** or ***bps,*** these are 1/100 of one percent. (1%=100bps). *So BMW's bonds would be offered at* ***"220bps over". 1%=100bps.***

Digest the terminology: Price of a bond cont'd

We are in October 2023, what is the value of the following investment?

An IBM Bond pays $115 every September 30 for the next 5 years. In September 2028 it pays an additional $1000 and retires the bond. **Question**: Can you tell me the face value of the bond?

If investors expect bond's YTM to be 7.5%

$$PV = \frac{115}{1.075} + \frac{115}{(1.075)^2} + \frac{115}{(1.075)^3} + \frac{115}{(1.075)^4} + \frac{1{,}115}{(1.075)^5}$$

$$= \$1{,}161.84$$

To express the price of the bond in PERCENTAGE TERMS:

Divide PV by the principal amount. Then multiply the answer by 100%:

PERCENTAGE PRICE = $\frac{\$1{,}161.84}{\$1{,}000} = 1.16184\ x\ 100\% = \mathbf{116.186\%}$

We say this bond has a price 116.18% and yield of 7.50%. Price is above 100%, so it is a ***premium bond,*** rated as **AAA. Research the term: credit rating agencies**

Bond rating agencies help addressing default risk

- Bond rating agencies assess the financial health of bond issuers
- Based on this assessment, bond rating agencies assign a rating to each bond
- The table below lists the possible bond ratings in declining order of quality

	Moody's	S&P's & Fitch
Investment Grade		
	Aaa	AAA
	Aa	AA
	A	A
	Baa	BBB
Junk Bonds		
	Ba	BB
	B	B
	Caa	CCC
	Ca	CC
	C	C

The highest quality bonds are rated triple-A.

Investment grade bonds have to be equivalent of Baa or higher.

Bonds that don't make this cut are called "high-yield" or "junk" bonds.

Again: Bonds are ***rated*** financial instruments

$\text{Price}_{bond} < \text{Face Value}_{bond}$ = discount bonds

The bond is paying an interest rate (Coupon Rate) which is lower than current rates (Yields/Returns) of similar risk investments. As a result, this bond is cheaper and its price is lower than the principal amount.

$\text{Price}_{bond} > \text{Face Value}_{bond}$ = premium bonds

The bond is paying an interest rate (Coupon Rate) which is higher than current rates (Yields/Returns) of similar risk investments. As a result, this bond is more expensive and its price is higher than the principal amount.

$\text{Price}_{bond} = \text{Face Value}_{bond}$ = par bonds

The bond is paying an interest rate (Coupon Rate) which is equal to current rates (Yields/Returns) of similar risk investments. As a result, this bond sells at PAR and its price is equal to the principal amount.

Digest the terminology: Price of a bond cont'd

$$PV = \frac{115}{1.075} + \frac{115}{(1.075)^2} + \frac{115}{(1.075)^3} + \frac{115}{(1.075)^4} + \frac{1{,}115}{(1.075)^5}$$

$$= \$1{,}161.84$$

We say this bond has a price 116.18% and yield of 7.50%

If the **US government** 5 - year bond had a YTM of 6.5%, how would you quote this IBM bond to an investor?

Digest the terminology: Price of a bond cont'd

Example - USA

In October 2023 you purchase a 3 - year US Government bond. The bond has an annual coupon rate of 4%, **paid semi-annually**. The **face value of the bond is $1,000**. If investors demand a 4.96% **annual** yield on 3 year investments, what is the value of the bond?

Questions:

How much is the coupon?

What is *r?*

$$PV = \frac{20}{1.0248} + \frac{20}{(1.0248)^2} + \frac{20}{(1.0248)^3} + \frac{20}{(1.0248)^4} + \frac{20}{(1.0248)^5} + \frac{1020}{(1.0248)^6}$$

$$= \$973.54$$

To find how to express the price of the bond in PERCENTAGE TERMS take the PV and divide it by the principal amount. Then multiply the answer by 100%:

PERCENTAGE PRICE = $\frac{\$973.54}{\$1{,}000} = 0.97354 \; x \; 100\% = \mathbf{97.354\%}$ **of face value**

Because this is a government bond we quote its yield in <u>absolute terms</u>, so we would say that the US government bond is offering 2.48% x 100 = 248 basis points semi-annually.

Some examples of bonds

Gilts = UK Government Bonds

Treasuries=US Government Bonds

T-Bonds-20/30years+ |T-Notes-10years or less| T-Bills-1year or less

Bunds = German Government Bonds

JGB= Japanese Government Bonds

Corporate Bonds = Issued by a corporation

High Yield Bonds == Low credit quality; issuer has high risk of default (junk bonds)

Some examples of bonds

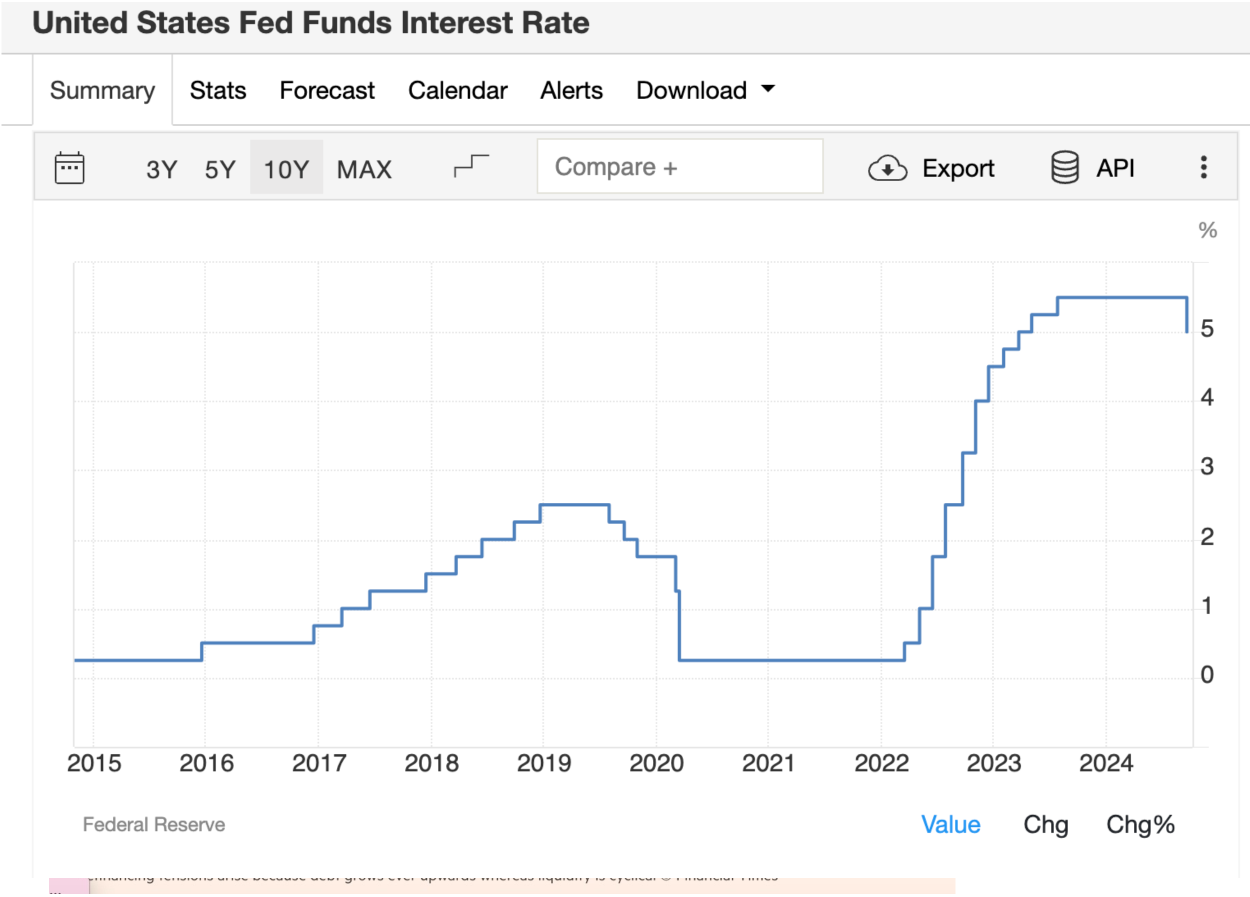
Secured Bonds (including mortgages) are backed by collateral (assets) if the borrower defaults. safety net, guarantee

Zero Coupon Bonds ("Strips") = pay no coupons (so, they are always priced at a discount: Recall my original bond I wanted to issue you!)

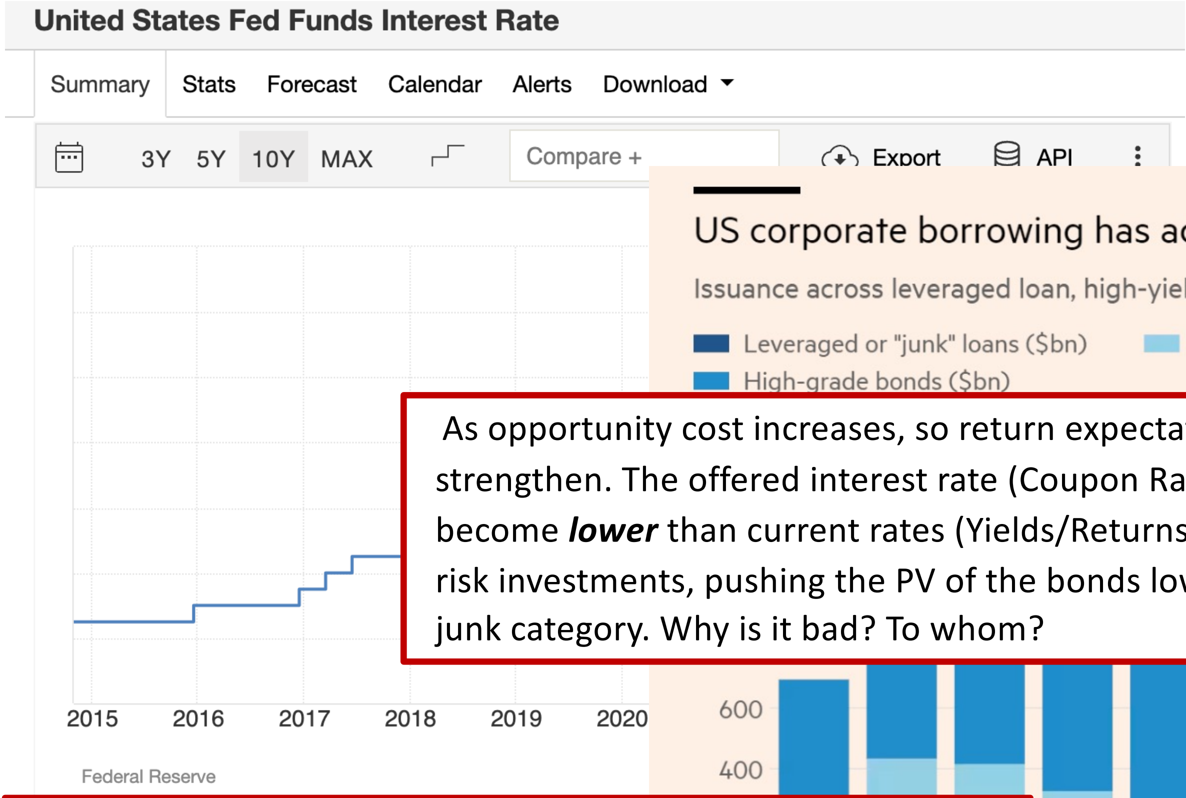
US and Canada style = semi-annual coupon (also Gilts, JGBs)

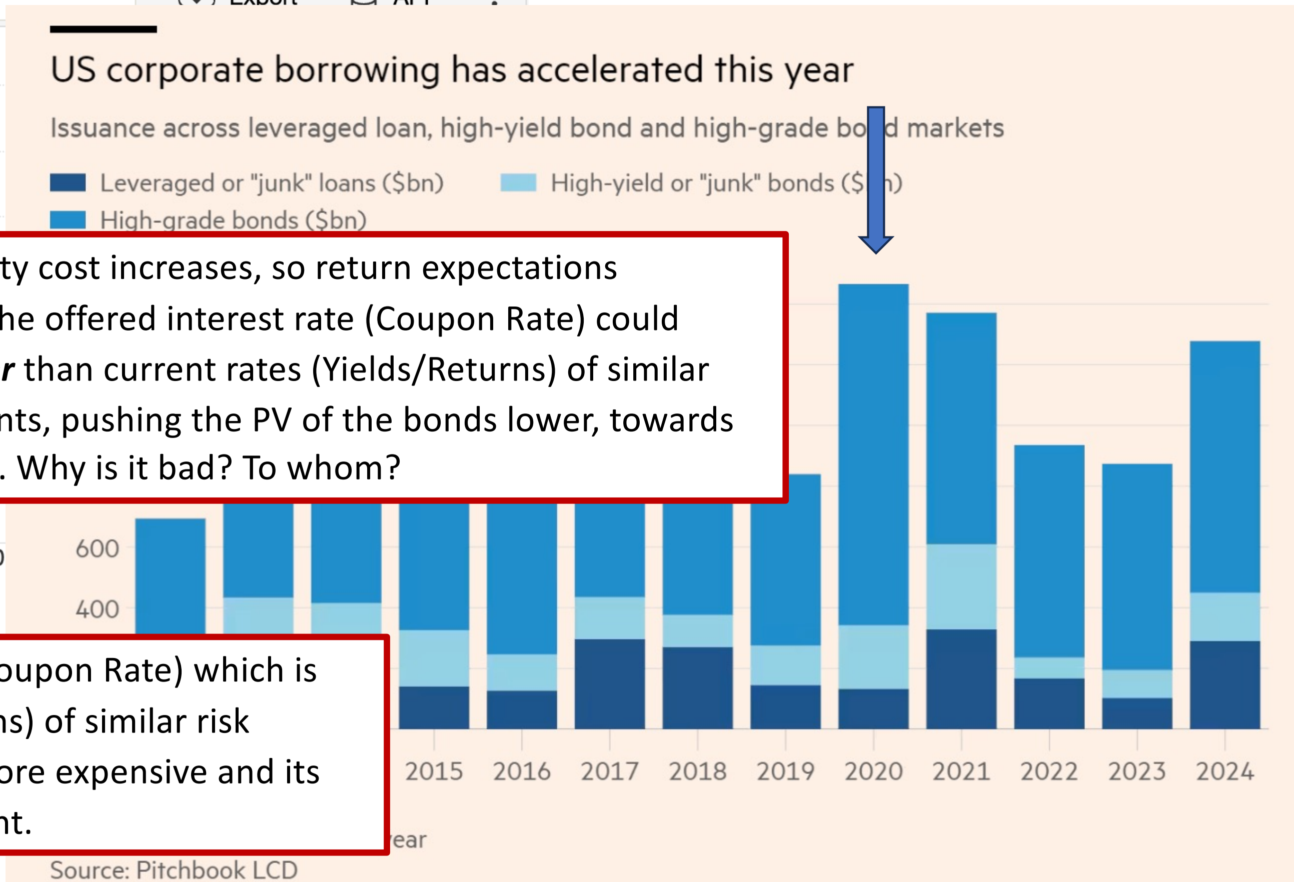
Eurobond style = annual coupon (most EU)

Setting interest rates carefully is the licence of central bankers



"… refinancing of those debts mostly taken out, a few years back, when interest rates were rock bottom."





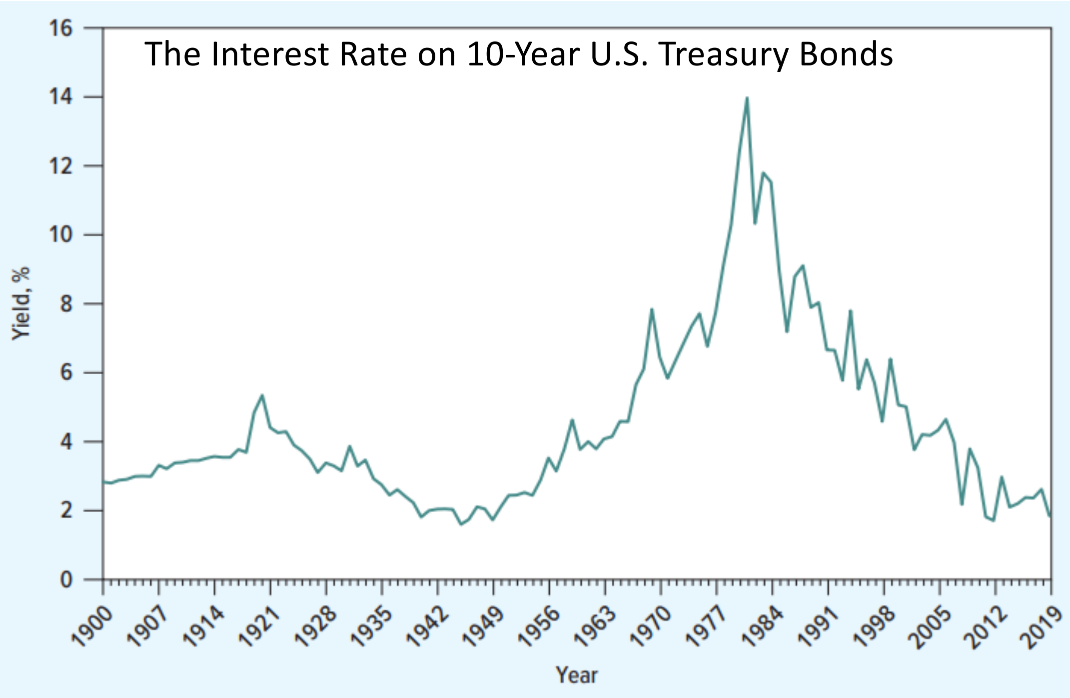
As opportunity cost increases, so return expectations strengthen. The offered interest rate (Coupon Rate) could become ***lower*** than current rates (Yields/Returns) of similar risk investments, pushing the PV of the bonds lower, towards junk category. Why is it bad? To whom?

Premium bonds pay an interest rate (Coupon Rate) which is higher than current rates (Yields/Returns) of similar risk investments. As a result, this bond is more expensive and its price is higher than the principal amount.

Investors’ demand regarding return, thus the yield of a bond might vary, and so, the bond price!

… bonds are financial assets, tradable, exposed to market dynamics.

As opportunity costs, so return expectations change, yield changes, thus the PV of the bond!



How would the bond price dynamics correspond?

$$PV = \frac{cpn}{(1+r)^1} + \frac{cpn}{(1+r)^2} + \ldots + \frac{(cpn + par)}{(1+r)^t}$$

$P_A > P_B \longleftrightarrow YTM_A < YTM_B$

Bond prices and yields **must** move to the opposite direction

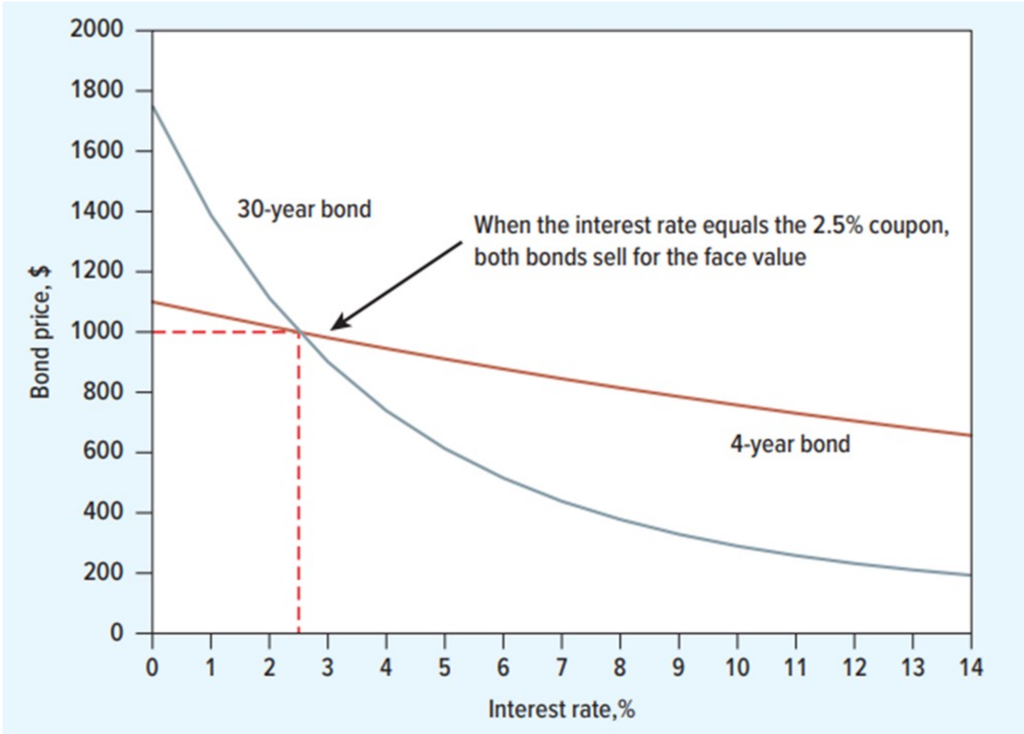
Not only rates, thus prices vary with interest rates.
Time to maturity matters, too!

See below the bond prices as a function of the interest rate. What can you conclude here?

Blue curve: 30-year bond
Brown line: 3-year bond

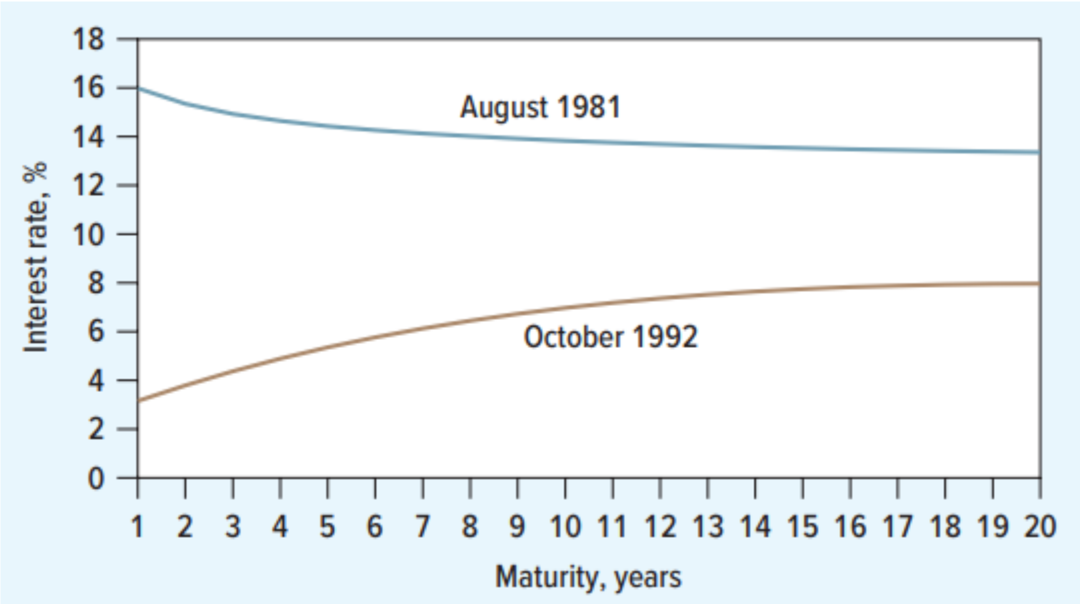
Price of long-term bonds is more sensitive to changes in the interest rate than is the price of short-term bonds.

Why?



Following the analogy, short and long-term interest rates exist, called the **term structure** of interest rates

… and **they do not always move in parallel**. In August 19 81, U.S. short-term interest rates were higher than long-term rates. In October 1992, the term structure was upward sloping and long-term rates were higher than short-term rates.



These curves that are plotted as a function of maturity are called the **yield curves**

… and the explanation for the dynamics of these curves relates to our own perceptions, biases, too, the way we predict the performance of the underlying economy!

Source: https://www.federalreserve.gov/data/nominal-yield-curve.htm.

Generally speaking: Seek for FED (Federal Reserve) data if you are interested in understanding the US market

We did a lot in this lecture, most importantly:

- Timing of payments matters
- Rates matter

- What is a bond? A promise!
  - This promise is constantly affected by market dynamics, thus makes sense to talk about its sensitivity, i.e. about its duration and yield structure. Stay tuned!