WELCOME TO
Introduction to Finance
for
MSc Financial Technology and Innovation

Lecture 9

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Last lecture we finished

- Valuation fundamentals in the context of risk
  - Portfolios, markets, financial assets (aka claims)

Last lecture we started

- Valuing a business
- Capital structuring decisions

Your toolset so far: Fundaments

- PV, FV
- Discount factor
- NPV, DCF
- Rate of Return rule
- Interest rates: simple or compounding

<u>Annual Percentage Rate (APR)</u>

$$\text{APR} = \text{rate per period} \times \text{periods per year}$$

<u>Effective Annual Interest Rate (EAR)</u>

$$\text{EAR} = (1 + \text{rate per period})^{\text{periods per year}} - 1$$

$$\text{PV} = \underbrace{\frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots + \frac{C_t}{(1+r)^T}}_{\textbf{DCF}}$$

$$\text{NPV} = C_0 + \sum_{t-1}^{T} \frac{C_t}{(1+r)^t}$$

$$\text{NPV} = \text{PV} - \text{required investment}$$

$$\text{Return} = \frac{\text{profit}}{\text{investment}}$$

$EAR = [1 + \frac{r}{m}]^m - 1$, where
$\boldsymbol{m}$ = period; $\boldsymbol{r}$ = rate

Your toolset so far: Series of payments

$$\text{PV of annuity}_{\text{t}}\text{: } C\times\left[\frac{1}{r}-\frac{1}{r\times(1+r)^{t}}\right]$$

$$\boxed{\text{PV of annuity}_{due} = C_1\times\left[\frac{1}{r}-\frac{1}{r(1+r)^{t}}\right]\times(1+r)}$$

- Annuity

$$\text{FV of annuity} = C\times\left[\frac{(1+r)^{t}-1}{r}\right]$$

- Perpetuity

$$\text{PV of perpetuity} = \frac{C_1}{r}$$

$$\boxed{\text{PV of perpetuity}_{due} = \frac{C_1}{r}\times(1+r)}$$

- Both their rates can grow, in fact(!)

$$\text{PV of perpetuity}_{\text{g=grow}} = \frac{C_1}{r-g}$$

$$\text{PV of Growing Annuity} =$$

$$= C_1\times\frac{1}{r-g}\times\left[1-\frac{(1+g)^{t}}{(1+r)^{t}}\right]$$

Your toolset so far: Bonds

- Price of a bond = PV of a bond $$\text{PV} = \frac{\text{cpn}}{(1+r)^1} + \frac{\text{cpn}}{(1+r)^2} + \ldots + \frac{(\text{cpn} + \text{par})}{(1+r)^t}$$

- Duration of a bond = weighted average(!) life of cashflows $$\text{Duration} = \frac{1 \times \text{PV}(C_1)}{\text{PV}} + \frac{2 \times \text{PV}(C_2)}{\text{PV}} + \frac{3 \times \text{PV}(C_3)}{\text{PV}} + \ldots + \frac{T \times \text{PV}(C_T)}{\text{PV}}$$

- Modified duration (since returns vary over time, so the cashflow) $$\text{Modified duration} = \text{volatility}\,(\%) = \frac{\text{duration}}{1 + \text{yield}}$$

Your toolset so far: Equities

- Understanding the return of a common stock

$$\text{Expected return} = r = \frac{\text{DIV}_1 + P_1 - P_0}{P_0} \quad \text{Expected Return} = r = \frac{Div_1}{P_0} + \frac{P_1 - P_0}{P_0}$$

- DDM

generalizing the one-period calculation +
defining a horizont

$$\text{Price} = P_0 = \frac{\text{DIV}_1 + P_1}{1 + r} \xrightarrow[\text{leads to}]{} P_0 = \frac{\text{DIV}_1}{(1+r)^1} + \frac{\text{DIV}_2}{(1+r)^2} + \ldots + \frac{\text{DIV}_H + P_H}{(1+r)^H}$$

- No growth

$$Perpetuity = P_0 = \frac{Div_1}{r} \; or \; \frac{EPS_1}{r}$$

- With growth (Gordon Growth Model)

$$PV_0 = \frac{C_1}{r - g}$$

Your toolset so far: Ratios for equity valuation

$$\text{Payout Ratio} = \frac{Dividend}{Earnings\ per\ Share\ (EPS)}$$

$$\text{Plowback Ratio} = \frac{Earnings\ per\ Share\ (EPS) - Dividend}{Earnings\ per\ Share\ (EPS)} = \frac{Amount\ Reinvested}{Earnings\ per\ Share}$$

Plowback Ratio+ Payout Ratio=1

$$\text{Return on Equity} = ROE$$

$$ROE = \frac{\text{EPS}}{\text{Book Equity Per Share}}$$

g = return on equity X plowback ratio

Your toolset so far: Measuring risk

Expected portfolio return for two stocks= $(w_1 r_1) + (w_2 r_2)$

Variance of returns = $\sigma^2 = \sum \frac{(Actual\ Return_n - Expected\ Return)^2}{n-1}$ $\xrightarrow{\text{Standard deviation}}$ σ

Covariance of returns (for two stocks): $cov_{1,2} = \rho_{12}\sigma_1\sigma_2$ $\xrightarrow{\text{Correlation coefficient}}$ $\rho_{12} = \frac{\sigma_{12}}{\sigma_1 x\, \sigma_2}$

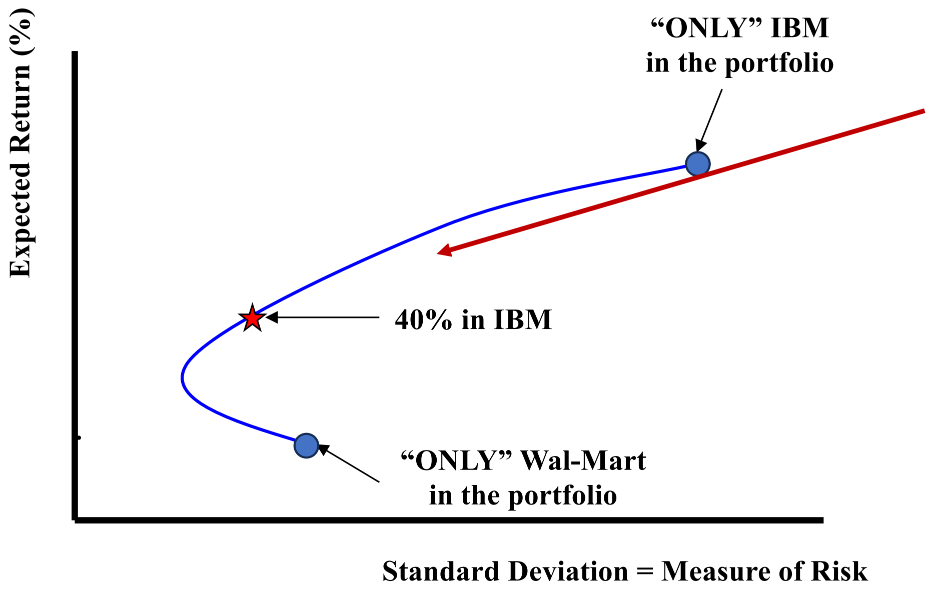
$$\text{Portfolio Variance} = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2(x_1 x_2 \rho_{12}\sigma_1\sigma_2)$$

Measure of systemic risk exposure of a stock (Beta): $B_i = \frac{\sigma_{im}}{\sigma_m^2}$

The effects of diversification is graphically represented by the **efficient frontier = constituting efficient portfolios based on different asset weights**

Key finding #1: Expected Returns and Standard Deviations vary given different weighted combinations of the two stocks

Key finding #2: Returns are dependent on the investment combinations that make up the portfolio.



The various weighted combinations of stocks on the blue line, i.e. curvature, constitute the set of ***efficient portfolios***.

These are portfolios that offer the least risk for a given level of return and the highest return for a given level of risk.

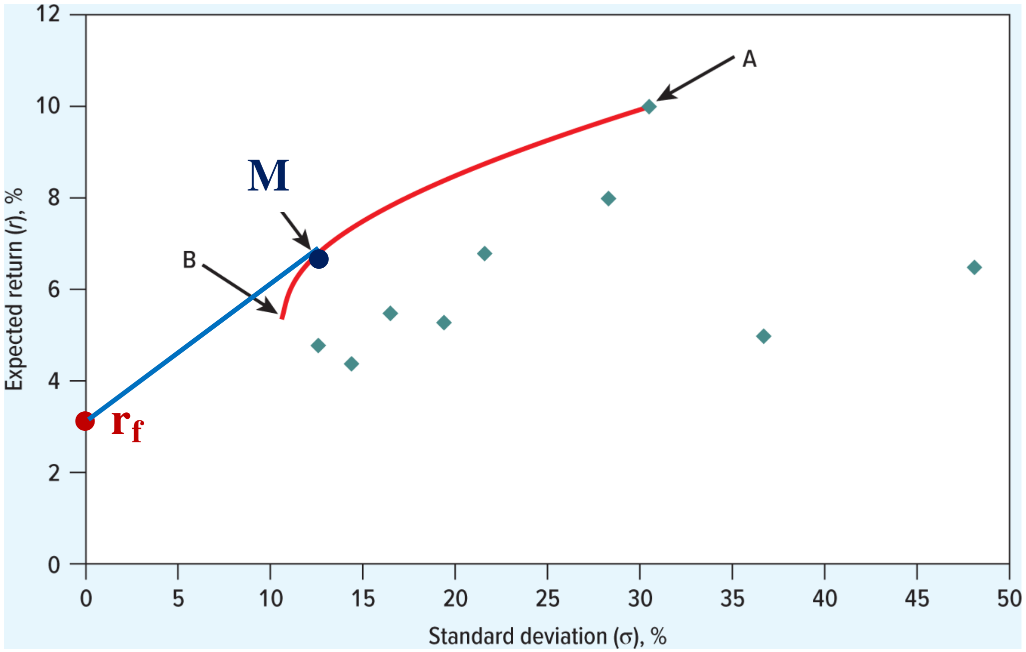
Rational investors are only willing to hold efficient portfolios.

What happens to our portfolio performance, if we decide to invest at the **risk-free rate?**

Claim: My efficient frontier would transform into a **straight line,** $f(r_f, M)$

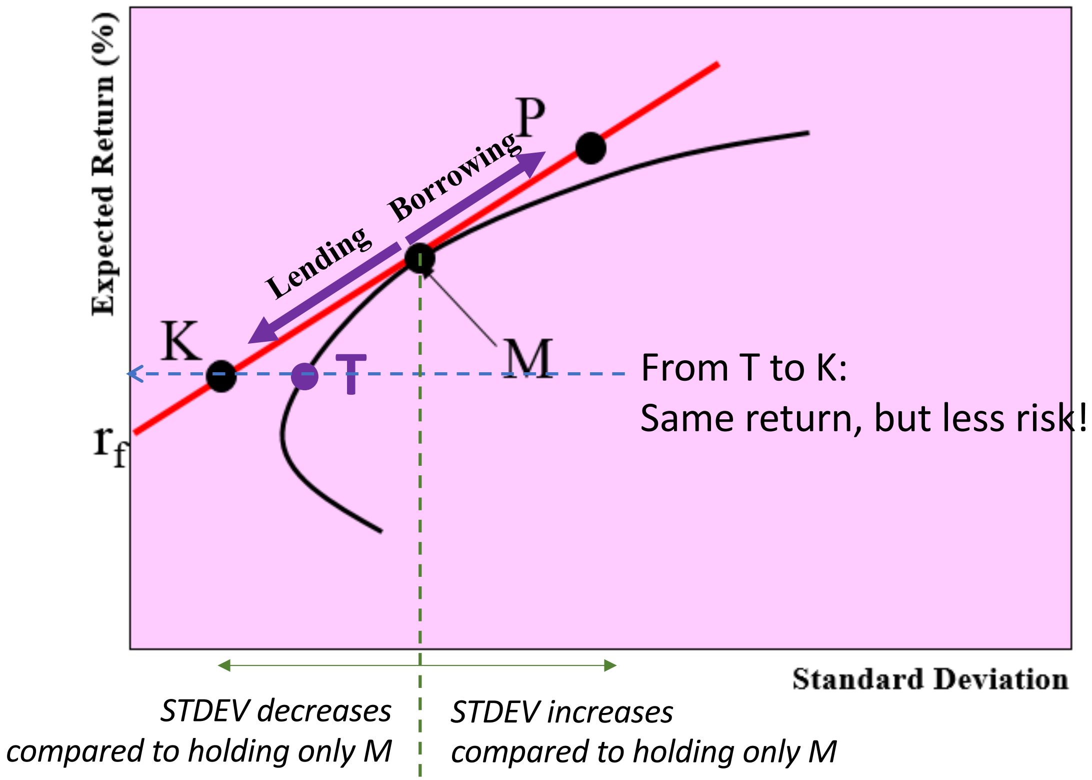
Why? $\boxed{\text{New Portfolio Variance} = x_1^2\sigma_1^2}$

It shows all the different possible combinations between portfolio M and the risk-free asset.



Consequently: Lending or Borrowing at the risk-free rate (r_f) allows us to "exist" outside the old efficient frontier

Investors can **improve** their risk and return position by lending (**K**) and borrowing (**P**), i.e. to "move" along the red line by changing the percentages invested in the risk-free asset and portfolio M.



Capital Asset Pricing Theory and Model (CAPM) describes, how the beta of an investment relates to its expected rate of return

Idea: Let M aggregate **all securities**, thus represent the market. Then, $f(r_f, M)$ must contain all possible portfolio combinations of a rational investor.

Do not forget: Beta can theoretically be between minus and plus infinite

$$\boxed{r_i = r_f + \beta_i \times \left(r_m - r_f\right)}$$

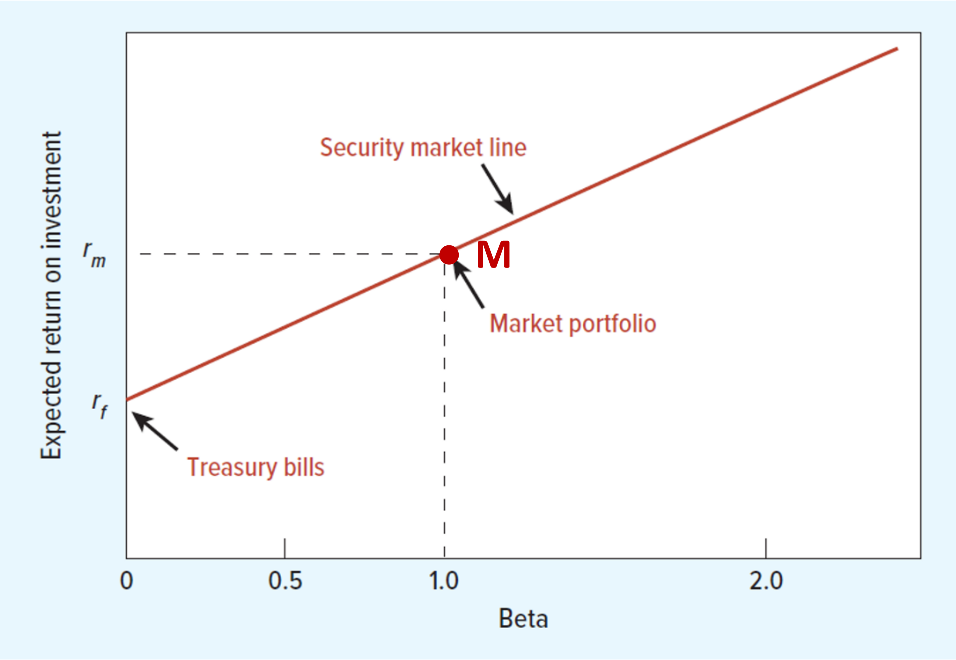
Where

r_i = investment's expected return

r_f = risk-free return

r_m = market return

r_m-r_f = market risk premium



If EMH holds, securities are "priced in" according to CAPM, since investors realize their return in the context of the risk they take

I am considering buying Marks & Spencer stocks. I assume the market return is expected to be, 8%, the YTM on T-Bills is 3%. Marks & Spencer's Beta is 1.2.

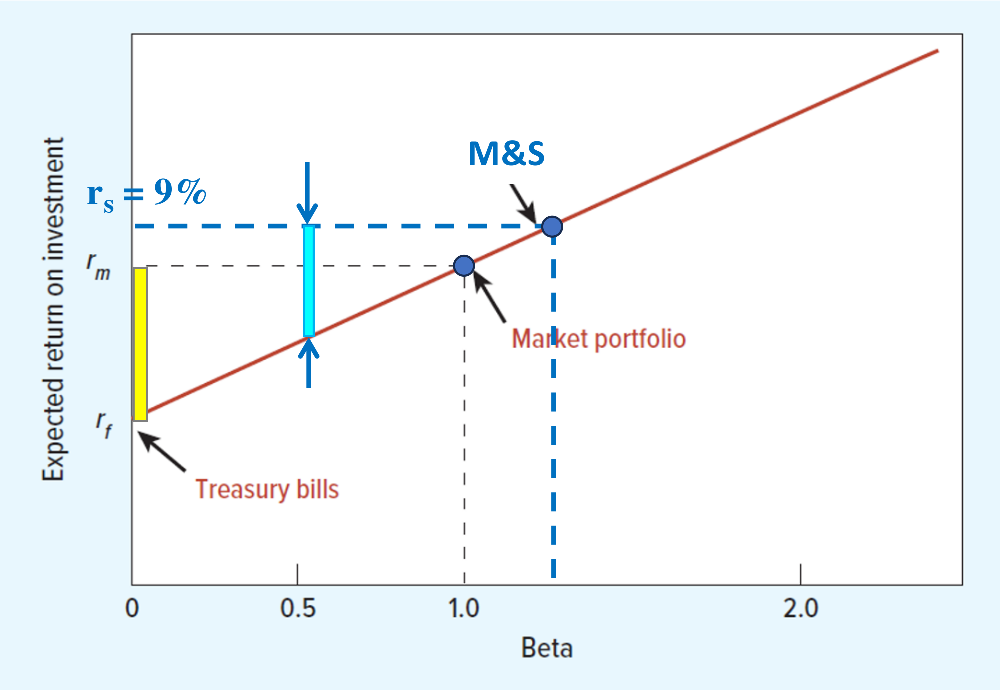
- **Question:** What is my expected return?

$$r_s = r_f + \beta\,(r_m - r_f)$$

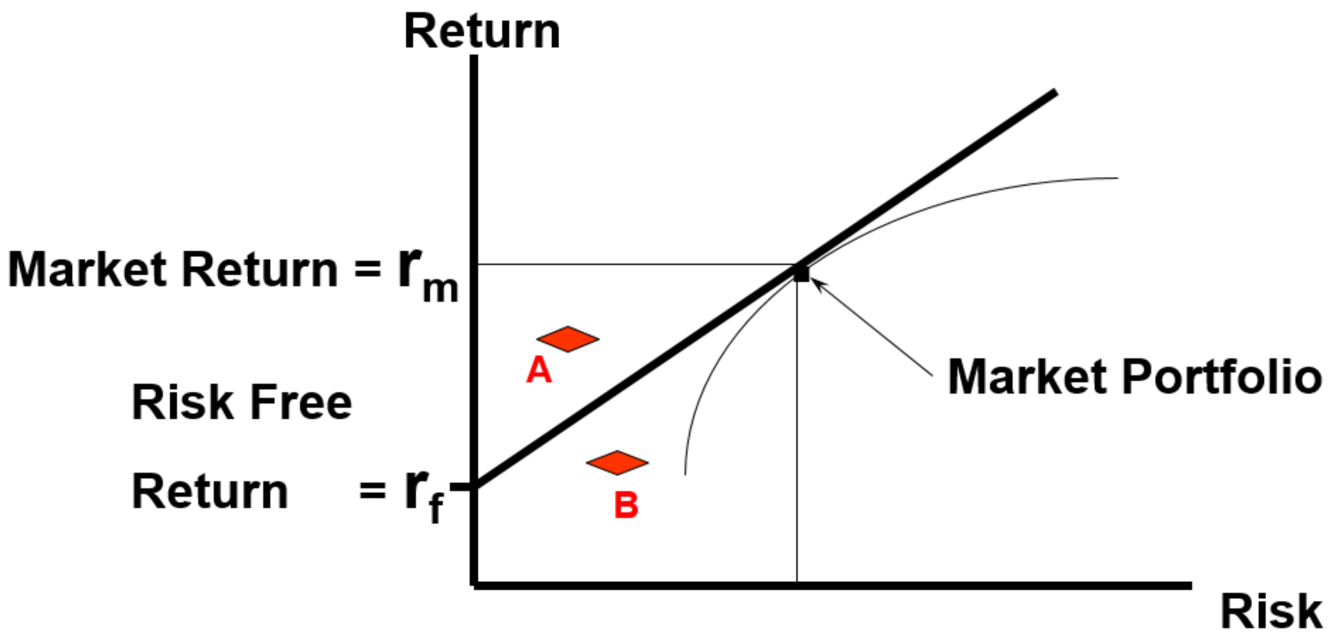
M&S = 3 + 1.2 (8-3) = 9 %

Investment's risk premium: 9% - 3% = 6%

Market risk premium: 8% - 3% = 5%



Shares A and B offer the risk and return indicated. Which one would you buy, which one would you sell?

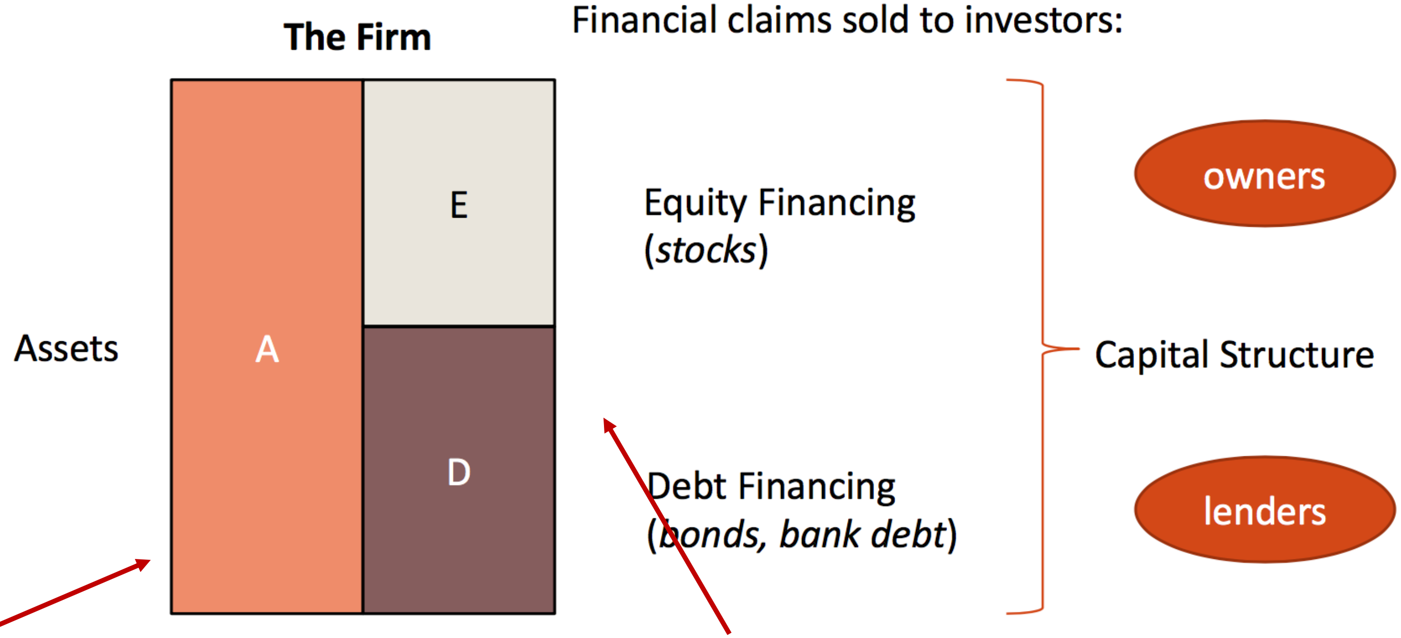


Markets as source of funding
Corporations raise money via financial claims

Recall the slides of Lecture1:

Real assets:
Contribute directly to my operation;
I (re)invest to mainitain it

Financial assets:
A contractual relationship, a promise I make to raise capital, and a promise I must keep



Deciding on real assets is a capital budgeting decision

Deciding on financial assets is a capital structuring decision

Valuing a Business in Practice

There are situations when we want to find the “true” value of the entire business

- Buy or sell entire businesses? “Pricing in” the stock for an IPO?

If a company is listed, we are “fine”, we use expected dividends per share for fair pricing.

If a company is not yet listed?

- We still follow the same logic but instead of using the expected dividends per share we immediately recall Principle3 and….
- Use the total free cash flow of the business

The free cash flow in any given year is the amount that’s left in the business after we have made all the necessary investments for growth

Every, so far studied calculation strategy for valuation would dictate that the “true value” we are looking for is the **sum** of the individual values of stages

Graphically, the red line represents the high-growth period, this is the value of the business during the foreseeable future.

In practice, a financial manager would calculate with a time horizon to limit the growth period.

Then, after a certain number of years the business reaches stability and maturity and we can assume that it will continue growing at this sustainable, stable growth rate forever.

It is also practical to assume this when we want to value a businesses as we cannot keep forecasting the company’s cash flows into eternity…!

We assume that the company will continue to grow at this stage in PERPETUITY.

Time horizon? Which formula for stock valuation contains a horizont?

Every, so far studied calculation strategy for valuation would dictate that the "true value" we are looking for is the **sum** of the individual values of stages.

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$$PV = \underbrace{\frac{FCF_1}{(1+r)^1} + \frac{FCF_2}{(1+r)^2} + \ldots + \frac{FCF_H}{(1+r)^H}} + \frac{HorizonValue_H}{(1+r)^H}$$

Valuation strategy of your business boils down to DCF-driven calculations, defining a feasible horizon

$$\boxed{PV = \underbrace{\frac{FCF_1}{(1+r)^1} + \frac{FCF_2}{(1+r)^2} + \ldots + \frac{FCF_H}{(1+r)^H}}_{\text{PV (First Stage of the Company's Life, this is the present value of the FCF that will be generated during the foreseeable future)}} + \underbrace{\frac{HorizonValue_H}{(1+r)^H}}_{\text{PV (Second Stage of the Company's Life, this is the present value of the business during the Stable/Sustainable/Constant growth rate, i.e. the horizon value)}}}$$

Let us walk through an example

Delicious Cakes has just paid a dividend of $5 per share. The dividends are expected to grow at **25%** per year for the **next three years** and at the rate of **10%** per year **thereafter**.

If the required rate of return on the stock is 18%, what is the current value of the stock?

- As a first step, let's determine the cash flows that we can expect from this company

(The ***free cash flow*** in any given year is the amount that's left in the business after we have made all the necessary investments for growth)

t=0, Div_0	t=1, Div_1	t=2, Div_2	t=3, Div_3	t=4, Div_4	t=5, Div_5	→
$ 5	$5 x (1+0.25)= $6.25	$5x(1+0.25)²= $7.81	$5x(1+0.25)³= **$9.77**	**$9.77**x(1+0.1)= $10.74	$10.74x(1+0.1)=$ 11.82	Dividends grow at 10% "forever"

from the **end of** Y3

Stage1: Rapid growth, r = 25%

Stage2: Sustainalbe growth "forever" with r = 10%

Let us walk through an example

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from the **end of** Y3

Stage-specific valuation strategy
FCF, **discounting for today with r=25%**

Stage-specific valuation strategy
Perpetuity (equities!) with r=10%, **but what is $HorizonValue_H$?**

Let us walk through an example

RoR on the stock = cost of capital(!) = 18%, g_1=25%, g_2=10%, what is the current value of the stock?

t=0, Div_0	t=1, Div_1	t=2, Div_2	t=3, Div_3	t=4, Div_4	t=5, Div_5	→
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from the **end of** Y3

Stage-specific valuation strategy
FCF, **discounting for today**

Stage-specific valuation strategy
Perpetuity (equities!) with r=10%, **but what is $HorizonValue_H$?**

To find the value of the expected dividends from the end of year 3 onward, we can use the **constant growth perpetuity** formula to **find the PV** of dividends during Stage 2
That will be our **Horizon Value.** There is a **growth expected** for the dividends for the second stage valuation, and the constant growth formula allows us reflecting on that.

$$PV_0 = \frac{Div_1}{r-g}$$

Let us walk through an example

RoR on the stock = cost of capital = 18%, g_1=25%, g_2=10%, what is the current value of the stock?

t=0, Div_0	t=1, Div_1	t=2, Div_2	t=3, Div_3	t=4, Div_4	t=5, Div_5	→
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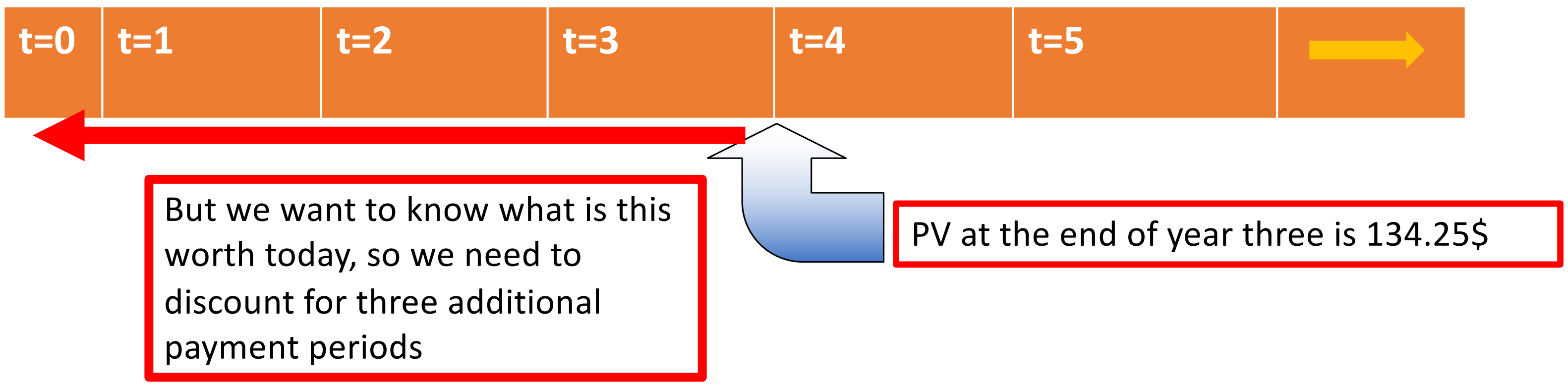
from the **end of** Y3

$$\text{Horizon Value} = PV_{End\ of\ Year\ 3} = \frac{Div_4}{r-g} = \frac{10.74}{0.18-0.10} = \$\ 134.25$$

But I want to find the value of the stock **today**, so **I need to discount the PV_3 value for another three payment periods** to be able to find the PV today at t=0

Let us walk through an example

RoR on the stock = cost of capital = 18%, g_1=25%, g_2=10%, what is the current value of the stock?



Thus, the PV_0 of cash flows during the Second Stage of company development is:

$$PV_{Stage\ Two} = \frac{PV_{End\ of\ Year\ 3}}{(1+r)^3} = PV_{End\ of\ Year\ 3}\ x\ \frac{1}{(1+r)^3} = \$134.25\ x\ \frac{1}{(1+0.18)^3} = \mathbf{\$\ 81.71}$$

Let us walk through an example

$ 81.71, r = 18%

t=0	t=1	t=2	t=3	t=4	t=5	

t=0, Div_0	t=1, Div_1	t=2, Div_2	t=3, Div_3	t=4, Div_4	t=5, Div_5	
$5	$5x(1+0.25)= $6.25	$5x(1+0.25)^2= $7.81	$5x(1+0.25)^3= $9.77	$9.77x(1+0.1)=$ 10.74	$10.74x(1+0.1)=$1 1.82	Dividends grow at 10% forever

To find the **total PV** we need to add the PV of cash flows expected during Stage 1, which takes place during the first three years of company life:

$$PV_{Stage\ One} = \frac{Div_1}{(1+r)^1} + \frac{Div_2}{(1+r)^2} + \frac{Div_3}{(1+r)^3} = \frac{\$6.25}{(1+0.18)^1} + \frac{\$7.81}{(1+0.18)^2} + \frac{\$9.77}{(1+0.18)^3} = \mathbf{\$\ 16.85}$$

Let us walk through an example

$ 81.71, r = 18%

t=0	t=1	t=2	t=3	t=4	t=5	

t=0, Div_0	t=1, Div_1	t=2, Div_2	t=3, Div_3	t=4, Div_4	t=5, Div_5	
$5	$5x(1+0.25)= $6.25	$5x(1+0.25)^2= $7.81	$5x(1+0.25)^3= $9.77	$9.77x(1+0.1)=$ 10.74	$10.74x(1+0.1)=$ 11.82	Dividends grow at 10% forever

$98.56

$$PV_{Stage\ One} = \frac{Div_1}{(1+r)^1} + \frac{Div_2}{(1+r)^2} + \frac{Div_3}{(1+r)^3} = \frac{\$6.25}{(1+0.18)^1} + \frac{\$7.81}{(1+0.18)^2} + \frac{\$9.77}{(1+0.18)^3} = \mathbf{\$\ 16.85}$$

+

$$PV_{Stage\ Two} = \frac{PV_{End\ of\ Year\ 3}}{(1+r)^3} = PV_{End\ of\ Year\ 3}\ x\ \frac{1}{(1+r)^3} = \$134.25\ x\ \frac{1}{(1+0.18)^3} = \mathbf{\$\ 81.71}$$

Thus, the current value of the stock, i.e. the fair price is **$98.56**

Buybook can increase share value, but as an artificial measure
Payout can decrease share value, but signalizes responsibility

	Assets	Liabilities	
Cash	£80		
		£80	Common Equity

After buyback, MyFirm has 80 shares outstanding, each worth 100p (=£80/80 shares!)

After the dividend payment, MyFirm has 100 shares outstanding, each worth 80p (=£80/100 shares)

Both cases leave £80 in cash and £80 in Equity
but the share buyback ALSO reduces the number of shares outstanding

Goal: To get everyone working together to maximize firm value

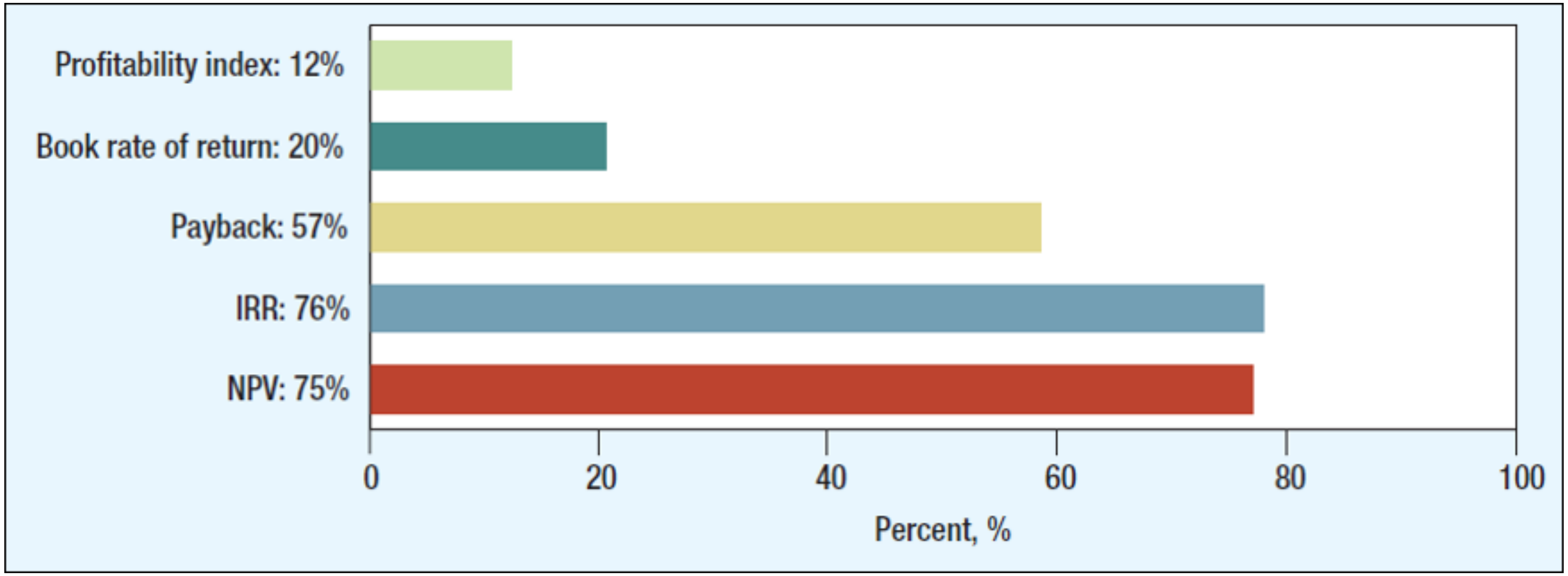
Shareholders = Owners
Principle

Managers = Employees
Shareholders' Agents

Capital budgeting (investment) decisions are "how to spend money" problems
Financing the capital investments are "how to raise money" problems

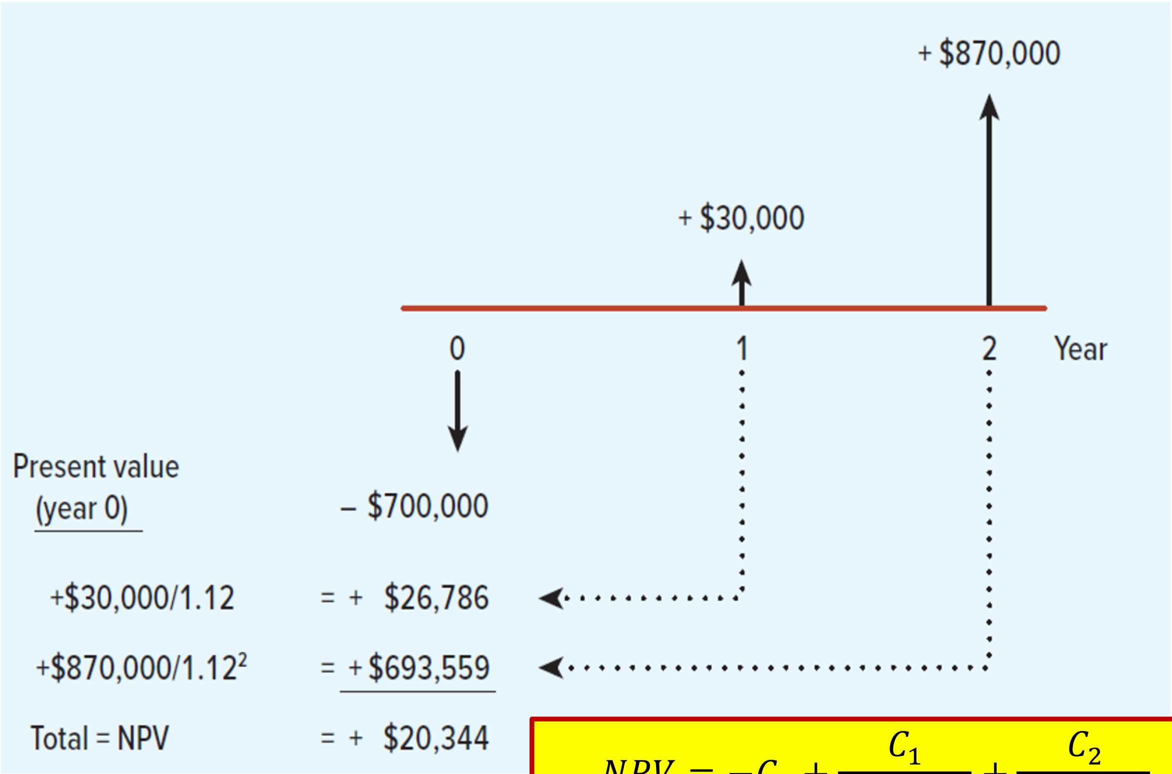
Financial managers should try as hard as they can to find and execute positive-NPV projects

CFOs' use of investment evaluation techniques



SOURCE: Graham and Harvey, "The Theory and Practice of Finance: Evidence from the Field," Journal of Financial Economics 61 (2001), pp. 187-243.

Maybe you recall this slide from Lecture2:



$$NPV = -C_0 + \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2}$$

In this case, you calculate with multiple cash-flows for each year

… and you do take care of the selected time reference (today)

$$\mathrm{PV} = \underbrace{\frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots + \frac{C_t}{(1+r)^T}}_{\textbf{DCF}}$$

$$\mathrm{NPV} = C_0 + \sum_{t-1}^{T} \frac{C_t}{(1+r)^t}$$

Book Rate of Return

Book Rate of Return (also called accounting rate of return)

- Average income divided by average book value over project life.

$$\boxed{\text{Book rate of return} = \frac{\text{book income}}{\text{book assets}}}$$

Managers rarely use this measurement to make decisions, as components reflect tax and accounting figures, not market values or cash flows:

- It doesn't consider the time value of money
- It doesn't consider the increased risk of long-term projects, and the increased uncertainty associated with long periods
- It doesn't consider the timing of the future cash flows

Payback focuses on break-even
"Short-term gain versus long-term pain"

Payback period: The number of years it takes before the cumulative forecasted cash flow equals the initial outlay, i.e. we reach break-even

- The payback rule says to **only accept** projects that "pay back" in the **desired time frame**

Problems:

- It ignores later-year cash flows
- It ignores the present value of future cash flows

Payback example, assuming 10% opportunity cost of capital

Examine the three projects and note the mistake we would make if we insisted on only taking projects with a payback period of 2 years or less.

Project	C_0	C_1	C_2	C_3	Payback Period	NPV@ 10%
A	- 2000	500	500	5000	3	+ 2,624
B	- 2000	500	1800	0	2	- 58
C	- 2000	1800	500	0	2	+ 50

Payback pushes firms to decide on an appropriate cut-off date. If it uses the same cut-off regardless of project life, it will tend to accept many poor short-lived projects and reject many good long-lived ones. This method ignores later year cash flows and the present value of future cash flows.

Recall: Rate of Return (RoR) rule: Invest in any project offering a rate of return that is higher than the opportunity cost of capital

$$\text{Rate of return} = \frac{\text{payoff}}{\text{investment}} - 1 \xleftarrow{\frac{\text{payoff} - \text{investment}}{\text{investment}}} \textit{Recall:}\ \text{Return} = \frac{\text{profit}}{\text{investment}}$$

Suppose an investment was initially made at $2000 and is now worth $2500.

Rate of Return for this investment is 25%: $\frac{2500 - 2000}{2000}$

… what is the problem here?

Time! What is the time-frame?

Internal (or Discounted Cash Flow) Rate of Return (IRR)

Internal Rate of Return (IRR) is the **discount rate** at which NPV = 0

Basically, we estimate the discount rate ("reverse interest rate") that helps addressing how much or **how well** the investment **performs** financially.

$$\boxed{\text{Rate of Return} = \frac{\text{Profit}}{\text{Investment}}}$$, where profit = payoff - investment

$$\text{Rate of return} = \frac{\text{payoff}}{\text{investment}} - 1 \quad NPV = -C_0 + \frac{C_1}{1 + discount\ rate} = 0 \rightarrow Discount\ rate = \frac{C_1}{C_0} - 1$$

Internal Rate of Return Rule:

Invest in any project if the opportunity cost of capital is **less** than the IRR.

You can purchase a turbo-powered machine tool gadget for $4,000.

The investment will generate $2,000 and $4,000 in cash flows for 2 years, respectively.

What is the IRR on this investment?

List what you know: 2 different cash-flow in, investment cost
List what you have: NPV, IRR rule

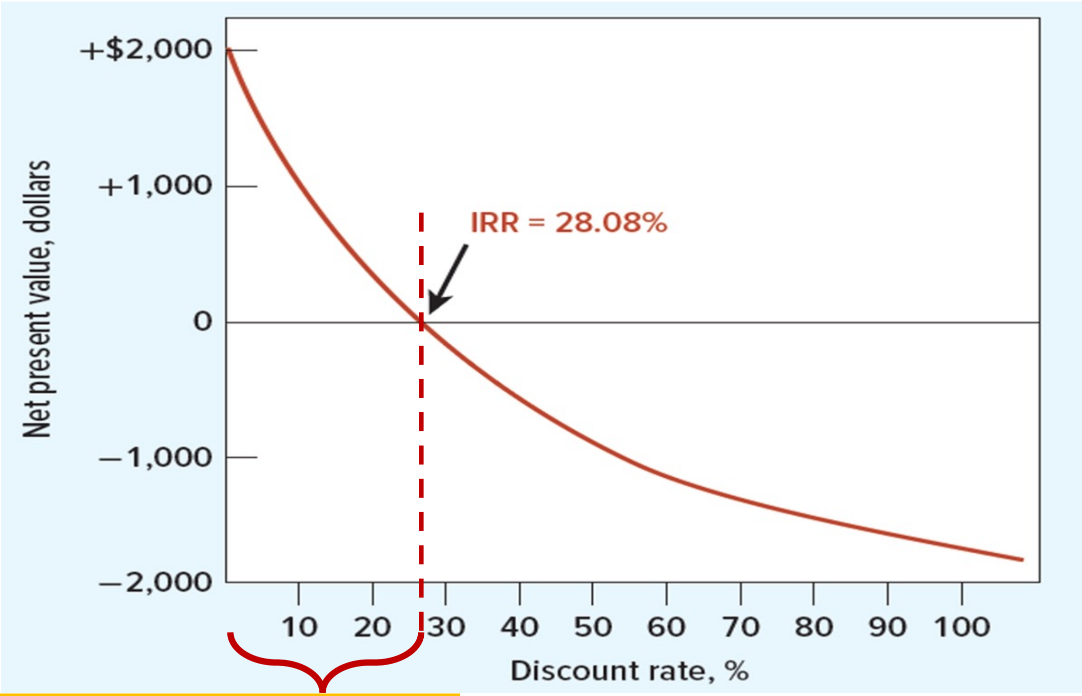
$$\text{NPV} = C_0 + \sum_{t-1}^{T} \frac{C_t}{(1+r)^t}$$

We are looking for the discount rate, i.e. for the IRR

$$\text{NPV} = -4,000 + \frac{2,000}{(1+\text{IRR})^1} + \frac{4,000}{(1+\text{IRR})^2} = 0$$

$$\text{IRR} = 28.08\%$$

Visualizing the solution
(we expect a declining exponential curve, crossing axe x only once)



Internal Rate of Return Rule:

Invest in any project if the opportunity cost of capital is **less** than the IRR.

How to calculate IRR?

$$\text{NPV} = C_0 + \sum_{t=1}^{T} \frac{C_t}{(1 + \text{IRR})^t} = 0$$

The complexity of this formula requires either a software (e.g. Excel) or a ***clever strategy***.

Any ideas? Through ***trial and error***

$$\text{NPV} = -4{,}000 + \frac{2{,}000}{(1 + \text{IRR})^1} + \frac{4{,}000}{(1 + \text{IRR})^2} = 0$$

$$\text{NPV} = -4{,}000 + \frac{2{,}000}{(1 + 0)^1} + \frac{4{,}000}{(1 + 0)^2} = 2.000$$

$$\text{NPV} = -4{,}000 + \frac{2{,}000}{(1 + 0.50)^1} + \frac{4{,}000}{(1 + 0.50)^2} = -889 \longleftarrow \text{IRR} = 28.08\%$$

FYI: IRR can be easily calculated with the Excel function IRR

Would IRR be sufficient to decide on your investment decisions? Not at all!

You are better off counting with both the NPV and IRR

$$\text{NPV} = C_0 + \sum_{t=1}^{T} \frac{C_t}{(1 + \text{IRR})^t} = 0$$

What do scenario A and scenario B represent? How do you explain the difference?

- HINT: What do the signs “tell” for Finance? How do you explain the misalignment between NPV and IRR?

Borrowing implies you need to pay back; lending implies that you receive your money back.

NPV reflects these two narratives, but IRR does not; after all: IRR is the rate.

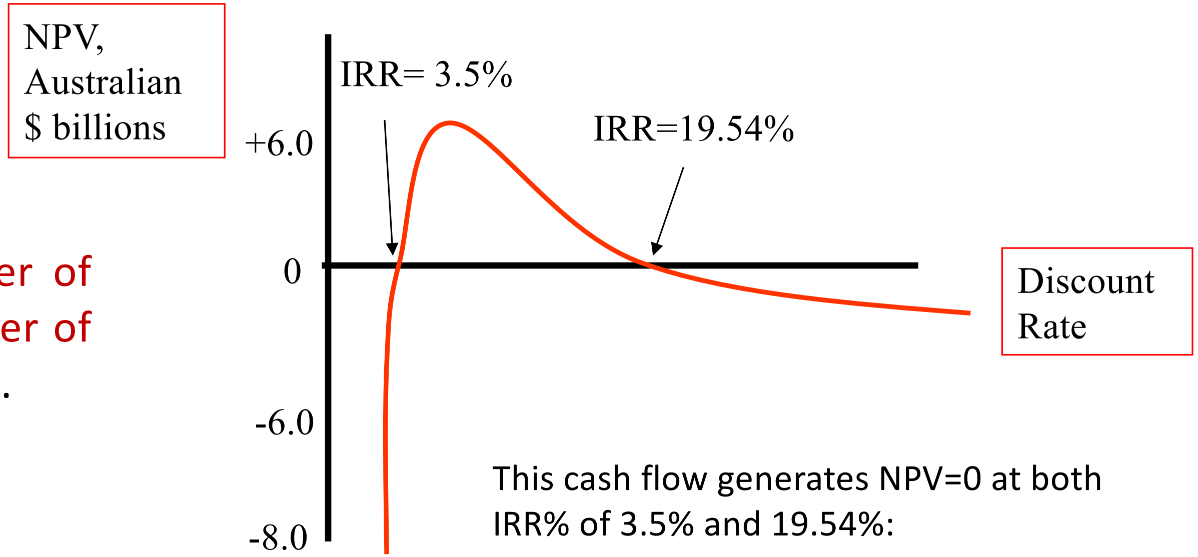
Project	C_0	C_1	IRR	NPV at 10%
A	−1,000	+1,500	+50%	+364
B	+1,000	−1,500	+50%	−364

IRR does not work with multiple rates of return either

Multiple Rates of Return implies that cash flows can generate NPV=0 at two different discount rates:

- Decommissioning and clean-up costs can sometimes lead to huge negative cash flows at the end of a project (as an example: Mining; nuclear waste management of power plants)
- Company needs to plan for late expenditures (hotels may receive a major face-lift, machine parts may need replacement)

The general rule is that the number of discount rates is equal to the number of changes in the sign of the cash flows.



This cash flow generates NPV=0 at both IRR% of 3.5% and 19.54%:

There are projects with no IRR and positive NPV

This is the case with the cash flows below, assuming 10% cost of capital:

Project	C_0	C_1	C_2	IRR	NPV @10%
C	$+1{,}000$	$-3{,}000$	$+2{,}500$	*None*	$+339$

There are mutually exclusive projects

Companies often decide upon several alternative ways of doing the same job.

- IRR sometimes ignores the magnitude of the project. Although project E requires a larger initial investment, it also generates a larger positive cash flow in year one.

Project	C_0	C_1	IRR	NPV @10%
D	$-10{,}000$	$+20{,}000$	100%	$+8{,}182$
E	$-20{,}000$	$+30{,}000$	$+75\%$	$+11{,}818$

Investment decisions, if resources are limited? Profitability Index (PI)!

We have four NPV Positive Projects, but not enough funds for all.

We must pick the projects that offer the highest NPV per dollar of initial outlay.

$$\text{Profitability Index} = \frac{\text{NPV}}{\text{Investment}}$$

$$\text{Profitability Index (A)} = \frac{21}{10} = 2.1$$

Cash Flows ($ millions)

Project	C_0	C_1	C_2	NPV @10%
A	-10	$+30$	$+5$	21
B	-5	$+5$	$+20$	16
C	-5	$+5$	$+15$	12
D	0	-40	60	13

Cash Flows ($ millions)

Project	Investment ($)	NPV ($)	Profitability Index
A	10	21	2.1
B	5	16	3.2
C	5	12	2.4
D	0	13	0.4

How about Project D? There is no investment!(?) Discount C_1 to find PV_0 and use as value of investment

We have four NPV Positive Projects, but not enough funds for all.

We must pick the projects that offer the highest NPV per dollar of initial outlay.

$$\text{Profitability Index} = \frac{\text{NPV}}{\text{Investment}}$$

$$\text{Profitability Index (A)} = \frac{21}{10} = 2.1$$

Cash Flows ($ millions)

Project	C_0	C_1	C_2	NPV @10%
A	-10	$+30$	$+5$	21
B	-5	$+5$	$+20$	16
C	-5	$+5$	$+15$	12
D	0	-40	60	13

Cash Flows ($ millions)

Project	Investment ($)	NPV ($)	Profitability Index
A	10	21	2.1
B	5	16	3.2
C	5	12	2.4
D	0	13	0.4

How about Project D? There is no investment!(?) Discount C_1 to find PV_0 and use as value of investment

$$\text{Profitability Index} = \frac{\text{NPV}}{\text{Investment}}$$

Discounting C_1 to find the PV in year 0: $PV_0 = \frac{-40}{(1+0.1)^1}$ = - 36.36

Cash Flows ($ millions)

Project	C_0	C_1	C_2	NPV @10%
A	−10	+30	+5	21
B	−5	+5	+20	16
C	−5	+5	+15	12
D	0	−40	60	13

Project	Investment ($) Absolute Value	NPV ($)	Profitability Index
A	10	21	2.1
B	5	16	3.2
C	5	12	2.4
D	36	13	0.4

Profitability Index allows you to rank the projects and decide upon your investments, based on your budget

The table below summarizes our findings from the previous example (in million$):

Proj	**NPV**	**Investment**	**PI**	**Rank**
B	16	5	3.2	1
C	12	5	2.4	2
A	21	10	2.1	3
D	13	36	0.4	4

Suppose we have only $10 million to invest

- Based on the ranking above we will invest these funds in projects B and C since they have the highest PI
- The combined NPV of the two projects is equal to NPV (B) + NPV (C) = 16 + 12 = 28

Remember: PI is particularly relevant when firms have capital constraint!

Thank you for reading the decks through!

Do not forget: This is just a start of the journey in order to familiarize yourself with the most important concepts, theorems and mechanics that you will be needing for your studies as well as for your professional life

Keep your references in mind:

FT FINANCIAL TIMES